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THESIS

AN ADAPTIVE COLLISION RESOLUTION PROTOCOL FOR RANDOM ACCESSED CHANNELS

by

Turhan Gurer

December 1985

Thesis Advisor: Jin-Fu Chang

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An Adaptive Collision Resolution Protocol for Random Accessed Channels

by

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Lieutenant Junior Grade, Turkish Navy
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Submitted in partial fulfillment of the
requirements for the degree of

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ABSTRACT

This thesis investigates the performance of two collision resolution protocols for a random accessed channel. The two proposed protocols are basically multislot collision resolution algorithms. In the first protocol, the number of slots opened is equal to the number of users involved in a collision. Each user randomly selects a slot. The second protocol is an adaptive version of the first protocol. Both of them are investigated numeric calculations and simulations.

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I. INTRODUCTION

A. DESCRIPTION OF THE PROBLEM

It is very common for users to access the same channel using the Time Division Multiple Access (TDMA) or Frequency Division Multiple Access (FDMA) techniques. These two methods provide multiple accessing, however there are disadvantages. For example reassigning of channel is difficult in FDMA. In TDMA guard times and headers reduce throughput. TDMA also requires central timing and synchronization control. In addition, these two methods are not suitable for handling interactive traffic such as that carried by computer networks. For this type of traffic, the ratio of peak load to average load is rather high. Users generating such traffic are often called bursty users.

An approach suitable for bursty users is to access the channel randomly. This is known RANDOM MULTIPLE ACCESSING (RMA or RA). In a random multiple access system each user has equal opportunity to access the channel when he has a message to send. Naturally, there is no coordination between users and collision is possible. Two or more messages trying to access the channel at the same time will lead to overlapping of messages which is called a collision. Overlapping of messages results in partial or complete destruction of each message and is not desirable. Collided messages need to be retransmitted until they are successfully received by their receivers. In the next section, we shall provide a brief historical review of random access techniques.

B. THE HISTORY OF THE RANDOM ACCESS TECHNIQUES

1. Pure ALOHA

In September 1968, the University of Hawaii [Ref. 1], began a research program to investigate alternatives to the use of conventional wire communications for computer-computer and console-computer links. The work was conducted by Norman Abramson and resulted in a system which is now known as the ALOHA system.

ALOHA was a system for remote stations on various islands of Hawaii to contact a central computer via a common radio broadcast channel. Stations use a very simple protocol for accessing the channel. This protocol is known as pure ALOHA.

The fundamental idea of pure ALOHA is very simple. Whenever a station or a user has a message it is transmitted immediately. No coordination is required for the user. Messages are first sent to the central station and then repeated back to all users in broadcast mode. In other words, the central station just acts as a repeater. Two possible outcomes may result from the transmission, success or collision. A success means the message does not overlap with other messages in time and will be successfully received by the receiver. Collision leads to destruction of all the messages involved in the collision. Therefore, after transmission of the messages the sender should monitor the feedback channel. If collision is detected, which can be done through parity checking, retransmission of the collided messages is required. In order to prevent unending collision, retransmission should be done in the following manner. Once a retransmission is necessary, a randomized delay is usually introduced before the retransmission actually takes place. Figure 1.1 shows of the execution of this protocol. In Figure 1.1 packets C and D collide during the first attempt.

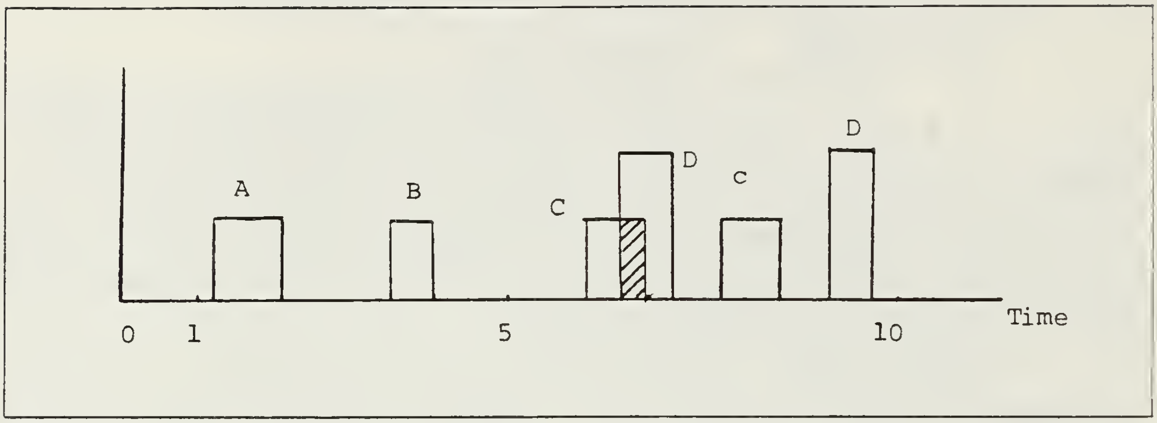

Figure 1.1 Pure Aloha

ALOHA protocol is its simplicity. A disadvantage is that it can only provide very low stable throughput. In this case, throughput is defined to be the average number of packets that can be successfully transmitted during one packet transmission time. A notation that has been generally accepted for throughput is S. Clearly $0 \le S \le 1$. In evaluating S, we usually assume an infinite number of users so that the channel input can be modeled as a Poisson process with mean arrival rate G packets/ τ sec. Here τ is the packet transmission time in seconds. Channel input includes newly generated and retransmitted packets. It can be proved that,

$$S = G \bullet \exp(-2G)$$

The maximum of S occurs at G=0.5 which results in a throughput 0.18. A system which can only achieve a maximum throughput if 0.18 is not efficient at all. One reason for this low efficiency is due to a 2τ second vulnerability period of a transmitted packet. This situation is depicted in Figure 1.2.

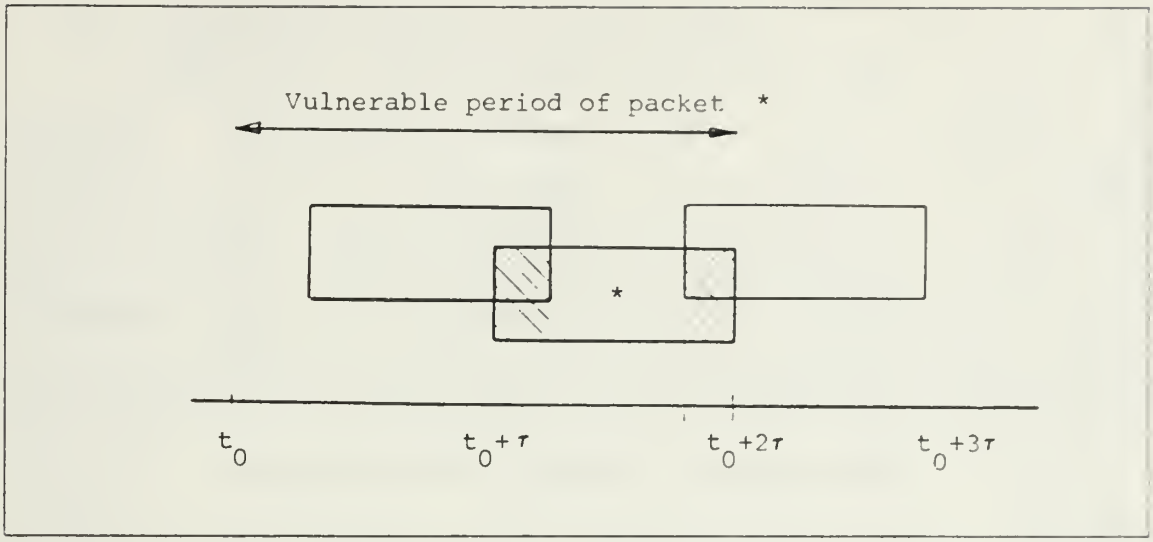

Figure 1.2 Vulnerability Period of the Shaded Packet

2. Slotted ALOHA

One way to improve the performance of the pure ALOHA system is to reduce the vulnerability period of the packet. In 1972, Roberts [Ref. 2], introduced the slotted ALOHA protocol for this purpose. The idea is demonstrated in Figure 1.3, in which the channel time axis is sliced into τ-second intervals called slots. Retransmission of data packets can take place only at the beginning of time slots as shown in Figure 1.3. Clearly, the vulnerability period of a packet is now τ seconds, which is one half of that of the pure ALOHA protocol. However the price we pay here is the requirement for strict synchronization among all the participating users. Once again, assuming an infinite number of users it can be shown that,

$$S = G \bullet \exp(-G)$$

The maximum throughput becomes 0.36 which is twice that of the pure ALOHA protocol. Figure 1.4 shows the throughput characteristics of pure and slotted ALOHA.

Although the system capacity, i.e., the maximum achieveable throughput is doubled by slotted ALOHA, the

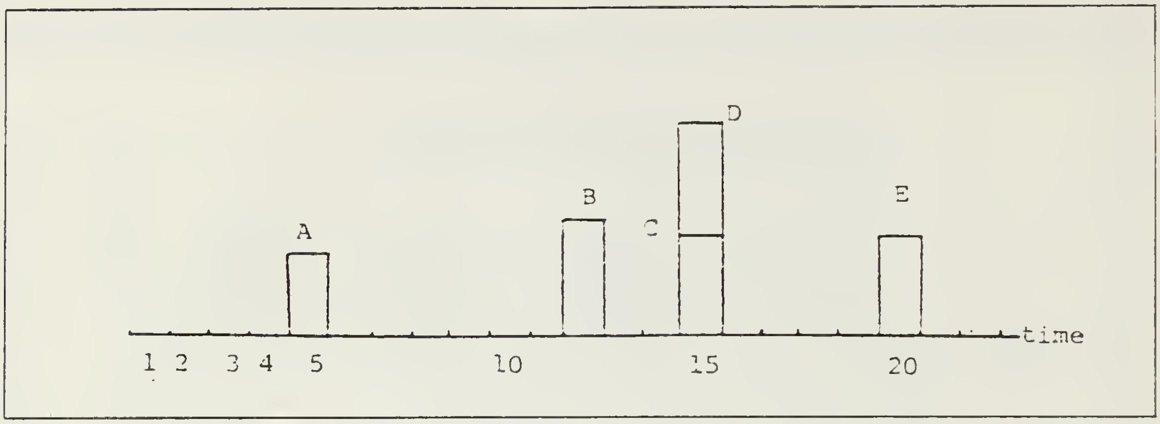

Figure 1.3 Slotted Aloha Channel

system is still not stable. If the mean arrival at G is pushed over 1.0 then there will be a drastic increase in the collision rate. This will eventually lead to a deadlock situation when the system throughput is reduced to zero.

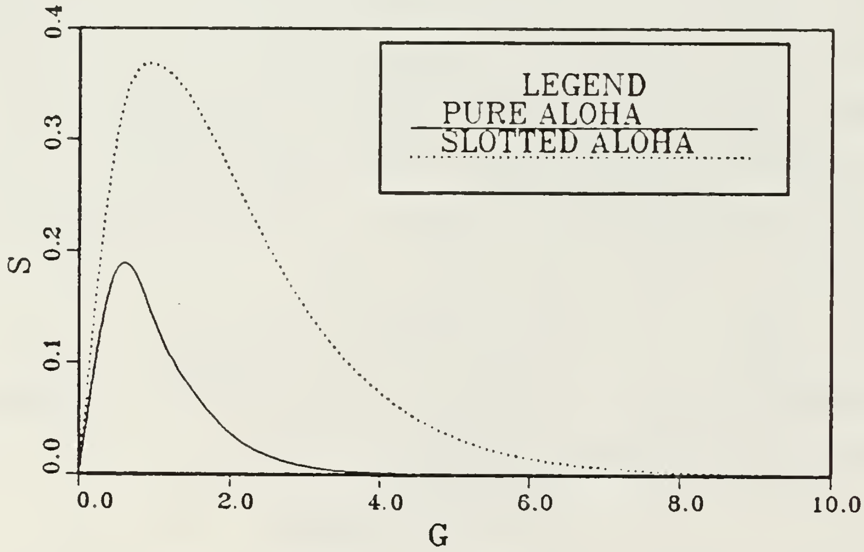

Figure 1.4 Aloha Systems

3. Collision Resolution Algorithms

Since the introduction of the ALOHA protocol by Abramson, tremendous efforts have been invested by researchers trying to enchance the performance of a random access system. Numerous papers have appeared as a result of these efforts. These recently have been centered around the collision resolution algorithm (CRA). A collision resolution algorithm is a protocol which is designed to reschedule the retransmission of those users involved in a collision so that each user eventually gets through. Usually before a collision is completely resolved transmission of new packet should not be attempted. The efficiency of a CRA is usually measured by the average number of time slots required to resolve a collision. This average number of time slots is usually called a collision resolution interval (CRI). Based on measurement one can usually derive the maximum achieveable throughput provided by the protocol. Almost all CRAs use a slotted system, i.e., the channel time is slotted.

Historically, the first CRA was proposed by Capetanakis [Ref. 3]. A similar approach was proposed by Hayes [Ref. 2]. Capetanakis' collision resolution algorithm (CCRA) was later improved by Massey [Ref. 4], by Gallager [Ref. 2] and by Humblet [Ref. 2]. An energy detector was also employed to improve the performance of the Capetanakis's protocol in [Ref. 3]. A thorough review of CRAs is provided in reference 2.

We use the example sketched in Figure 1.5 to illustrate Capetanakis's idea. Here the time axis is organized into 2 τ-slot frames. At the termination of the previous collision resolution interval all of the users are waiting for transmission of their packets. Each one randomly picks a slot in the next frame and transmits his packet. In Figure 1.5 we have a total of five users trying to access the channel. They are represented by A, B, C, G and H. Users A,

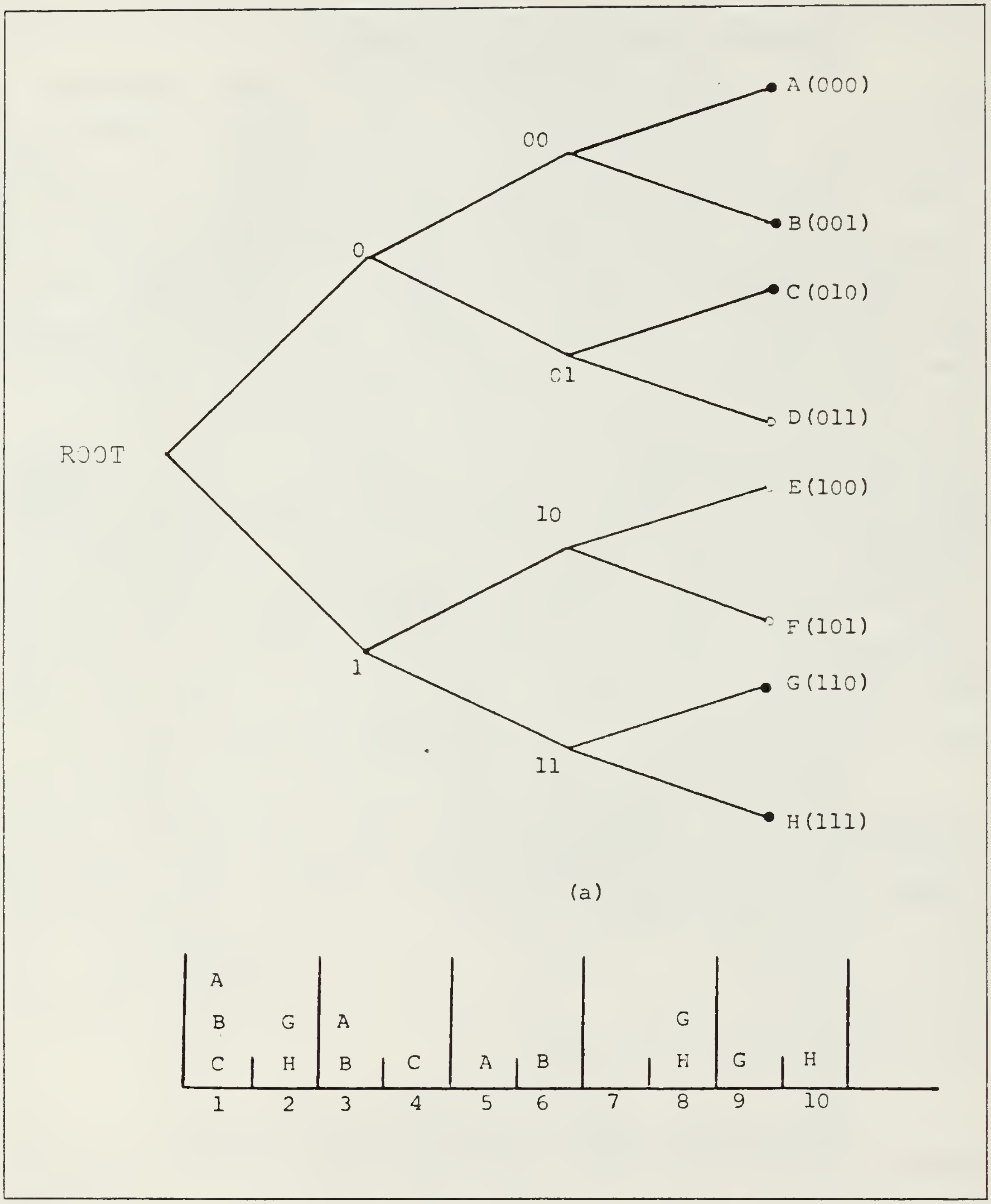

Figure 1.5   Binary Tree Algorithm

B and C select the left slot, while G and H use the right slot. Of course collision occurs in both slots. Slot status information available to users is ternary, i.e.,empty, success or collision. Since a collision is detected in slot 1, we then proceed to resolve the collision. We let users involved in the collision of slot 1, independently and randomly select slot 3 or 4 and retransmit. By this we randomly divide A, B and C into two groups. On our example A and B select slot 3, and C uses slot 4. Since C is now a success, i.e., then proceed to resolve the collision between A and B. Collision between G and H can be resolved only when A, B, and C have been successfully transmitted. A tree representation of Capetanakis protocol is given in Figure 1.5(a).

Tree Algorithm : Each leaf on the tree represents a source and each source has three bits binary address in this example. It might be 8 sources in the tree. There are five active sources in the Figure 5(a). The root of the tree defines all collision resolution interval, First upper branch leaves go to the left slot of the first group (A, B, C in the slot 1). Lower branch leaves go to the right slot of the first pair (G, H in the slot 2).The system shows the retransmission of the A, B, C in the second slot pair, C is alone and means success. After retransmission packet A and B pick up slots 5 and 6. Up here CCRA finished the upper branch collisions. Next step is to retransmit G and H by using the same way.

Capetanakis provides two versions of his protocol, Static and Dynamic. The maximum achieveable throughputs are 0.346 and 0.430, for static and dynamic CCRA respectively.

Later, Massey improved Capetanakis's algorithm by observing that if a collision is followed by an empty slot, one slot can be saved by repeating the random retransmission. This modification allowed a throughput of 0.46.

Gallager introduced a different conflict resolution algorithm with guaranteed stability and had first come first served characteristics. He selects an initial arrival interval I. It performs subsequent subdivisions of I whenever collision occurs, and it reinitializes whenever a collision occurs within the first I subdivisions. Gallager obtained a maximum throughput of 0.4872. Humblet improved Gallager's algorithm and increased the throughput to 0.48775.

In 1982, Georgiadis and Papantoni-Kazakos [Ref. 5] proposed a collision resolution algorithm called (CRAI). CRAI employs a device called an energy detector to tell the number of the users involved in a collision. Based on this information, they suggest an optimal way of dividing users into two groups once a collision involving these users is detected. Their protocol offers a maximum channel throughput of 0.53. The difficulty with this protocol is that it assumes the availability of an infinite energy detector.

C. THE SCOPE OF THIS THESIS

In this thesis we examine two new protocols which are not only easy to implement but also offer satisfactory performance. In Chapter II, we study the MSCRA protocol where each, time we open a number of time slots and let users involved in the collision randomly choose one slot and retransmit. The number of slots opened equals the number of users involved in the collision. Therefore, we have to assume the availability of an infinite energy detector. Chapter III gives an adaptive version of this protocol. Finally, the conclusion is presented in Chapter IV.

II. MULTISLOT COLLISION RESOLUTION ALGORITHM

A. INTRODUCTION

Suppose we have an energy detector of infinite capacity so that when a collision occurs, we know exactly how many users are involved in the collision. After the detection of a collision involving n users, we immediately open the next m slots for collision resolution. We let each of the collided users pick one of these m slots randomly for retransmission, and " m " should be selected to increase the average number of successful transmissions in the selected m slots and to maximize throughput. Towards this goal, we shall now prove that the optimum m happens to be n (n is the number of the collided users).

Suppose we wish to distribute n packets into m slots.

X = The random variable representing the number of the slots which contain exactly one packet when distributing n packets into m slots. Physically X represents the number of successes.

Next let $E[X]$ denote the expectation of X. Then we have the following special cases.

Case 1 : $n=1 \quad m>1 \quad ; \quad E[X] = 1$

Case 2 : $n=2 \quad m=2$; It is impossible to have only one success in this case. Distributing two packets into two slots, which is equivalent to throwing two balls randomly into two boxes, always results in either two successes or no success at all. In other words,

$$E[X] = 2\cdot(1/2) + 0\cdot(1/2) = 1$$

Case 3 : n=2 m=3

$$E[x] = 2 \binom{m}{1}\binom{m-1}{1} / m^2 = 2\binom{3}{1}\binom{2}{1} / 9 = 1.33$$

Case 4 : n=2 m≥2

$$E[X] = 2 \binom{m}{1}\binom{m-1}{1} / m^2 = 2m(m-1)/m^2$$

The denominator m^2 represents the number of ways to throw 2 balls into m boxes. The numerator tells the number of ways of select two distinct boxes from m boxes so that each box containing exactly one ball. Obviously E[X] is a function of m. To find the m which maximizes E[X] we perform

$$\frac{d}{dm}\left[\frac{E[x]}{m}\right] = \frac{d}{dm}\left[\frac{2(m-1)}{m^2}\right] = \frac{2m-2m(2(m-1))}{m^3} = 0$$

which leads to m=2

Case 5 :

n=3 m=3 In this case we never can have exactly two successes when distributing 3 packets into 3 slots. If the number of successes is two, the third packet must also be a success. Therefore we can have either one or three successes. Thus,

$$E[X] = 1\left[\binom{3}{1}\binom{3}{1}\binom{2}{1}\binom{2}{2} + 3\binom{3}{1}\binom{2}{1}\binom{1}{1}\right] / 27 = 1.33$$

Case 6 :

$n=3 \quad m>3$

$$E[X] = \left[1\binom{m}{1}\binom{n}{1}\binom{m-1}{1}\binom{n-1}{2}+3\binom{m}{1}\binom{m-1}{1}\binom{m-2}{1}\right] / m^n = \frac{3m(m-1)^2}{m^3}$$

In order to find optimal m, again we carry out

$$\frac{1}{3}\frac{d}{dm}\left[\frac{E[X]}{m}\right] = \frac{d}{dm}\left[\frac{(m-1)^2}{m^3}\right] = \frac{m^3 2(m-1)-(m-1)^3 m^2}{m^6} = \frac{(m-1)(3-m)}{m^4} \tag{2.1}$$

which yields $m=3$

In general, it can be easily seen that

$$E[X] = \frac{nm(m-1)^{n-1}}{m^n} = \frac{n(m-1)^{n-1}}{m^{n-1}}$$

If we let m approach infinity, the l'Hopital's Rule shows

$$\lim_{m\to\infty} E[X] = \lim_{m\to\infty} \frac{n(m-1)^{n-1}}{m^{n-1}} = \lim_{m\to\infty} \frac{n(n-1)(m-1)^{n-2}}{(n-1)m^{n-2}}$$

$$= \lim_{m\to\infty} \frac{n(m-1)^{n-2}}{m^{n-2}} = n$$

which is intuitively reasonable, for the optimal selection of n we have.

$$\frac{1}{n}\frac{d}{dm}\left[\frac{E[X]}{m}\right] = \frac{(n-1)(m-1)^{n-2}m^{n}-nm^{n-1}(m-1)^{n-1}}{m^{2n}}$$

$$= \frac{m^{n-1}(m-1)^{n-2}(-m+n)}{m^{2n}}$$

Which results in m=n.

B. DESCRIPTION OF THE MSCRA

First, we outline the fundamental assumptions to be used in this chapter.

1. Traffic generated by network users collectively form a Poisson process.
2. Channel time is slotted.
3. An energy detector of infinite capacity assumed in reference 3 is available.
4. There are no new packet transmissions until the collision has been resolved.
5. There is no propagation delay,
6. There is no channel errors.

Based on our discussion in Section II.A the MSCRA can be stated as follows. After a collision involving n users the next n slots are opened for retransmission. Each user randomly selects one of the n slots and retransmits. At the end of the n slots if i of the users have successfully transmitted their packets, i.e., n-i of them are still involved in collisions, the process is repeated with n-i

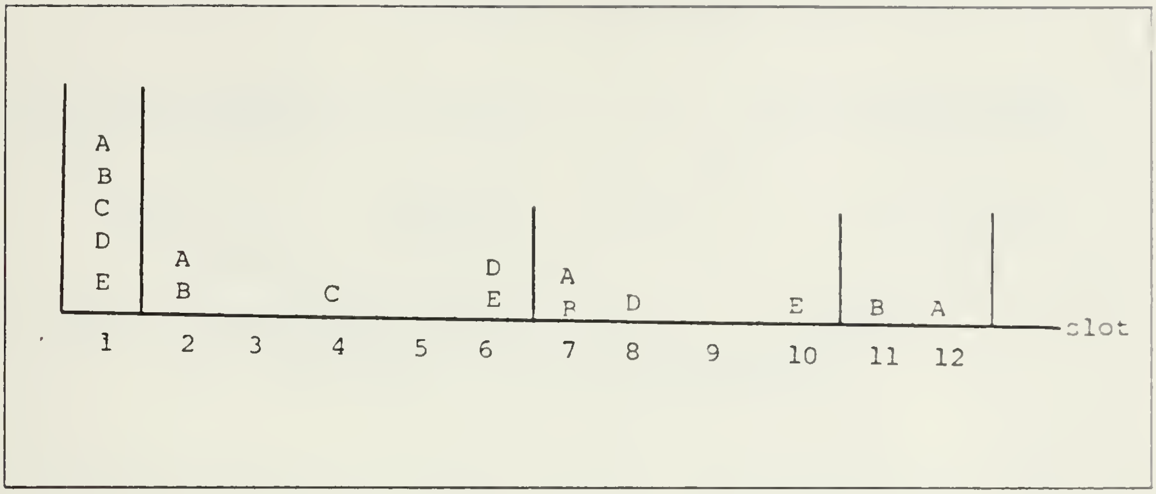

Figure 2.1 Multislot Collision Resolution Algorithm

In this example, the energy detector shows 5 collided packets, at the very beginning and the system opens the next five slots for collision resolution. During this five-slot period there is only one success and four packets still collided. The system then opens four more slots. Now A and B are in collision while D and E succeed. It is thus necessary to open another two slots (slot 11 and 12). In our example A and B are now successes.

It can be seen from this example that after each retransmission, the number of successes and still colliding packets can be assertained. The number of yet to be opened slots is always equal to the number of the packets remaining to be resolved. In this example, the length of the CRI (Collision Resolution Interval) happens to be 12 slots.

C. ANALYSIS OF THE MSCRA

In this section we begin by finding the probability of having i successes in distributing n balls into m boxes. i.e., P(X=i).

if n=1 and m=1 then obviously $P(X=1) = 1$

if n=2 and m=2 then $P(X=0) = \binom{2}{1}\binom{2}{2} / 2^2 = 0.5$

$P(X=1) = 0$

$P(X=2) = \binom{2}{1}\binom{2}{1}\binom{1}{1}\binom{1}{1} / 2^2 = 0.5$

if n=3 and m=3 then

$P(X=0) = \binom{3}{1}\binom{3}{3} / 3^3 = \frac{1}{9}$

$P(X=1) = \binom{3}{1}\binom{3}{1}\binom{2}{1}\binom{2}{2} / 3^3 = \frac{2}{3}$

$P(X=2) = 0$

$P(X=3) = \frac{3}{3^3} = \frac{2}{9}$

In resolving a collision involving n users, we define the system state to be the number of collided packets remaining to be resolved. Figure 2.2 shows the state transition diagram in resolving n collided packets. The transition from state n to state i is labeled by a transition probability $P_{n,n-i}$. Thus $P_{n,n-i}$ represents the probability of having n-i success in an n slot period.

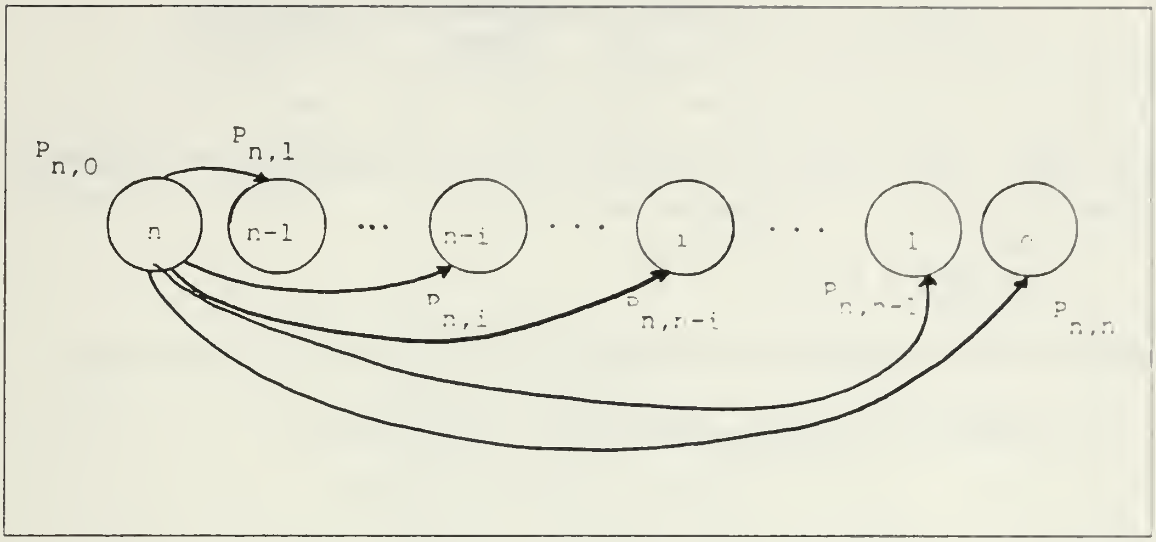

Figure 2.2 State Transition Diagram of An n slot Period

An important performance measure of the protocol is $\bar{L}_n$ the expected number of slots required to resolve a collision involving n users. From Figure 2.2.

$$\bar{L}_n = P_{n,0}\bar{L}_n + P_{n,1}\bar{L}_{n-1} + \dots + P_{n,n-1}\bar{L}_1 + P_{n,n}\bar{L}_0 \qquad (2.2)$$

Where $\bar{L}_0 = \bar{L}_1 = 0$

Since it is not possible to have exactly n-1 successes, we have

$$P_{n,n-1} = 0$$

Rewriting Eqn (2.2)

$$\bar{L}_n = n + \sum_{i=0}^{n} P_{n,i}\bar{L}_{n-i} = n + P_{n,0}\bar{L}_n + \sum_{i=1}^{n} P_{n,i}\bar{L}_{n-i}$$

We obtain,

$$\bar{L}_n = \frac{1}{1 - P_{n,0}} \left[n + \sum_{i=1}^{n} P_{n,i} \, L_{n-i} \right] \qquad (2.3)$$

Finally, the expected number of slots required in transmitting a total of n packets is,

$$\bar{L}^d_n = \bar{L}_n + 1$$

Where the 1 is due to the slot of the initial collision.

P can be obtained recursively as follows,

$$P_{n,i} = A^n_{n,i} \;/\; n^n$$

Where

$A^n_{n,i}$ = The number of possible arrangements in distributing n packets into n slots, which result in i successes.

$$A^n_{n,i} = \binom{n}{i}\binom{n}{i} i! \left[(n-i)^{n-i} - A^{n-i}_{n-i,1} - A^{n-i}_{n-i,2} - \dots - A^{n-i}_{n-i,n-i} \right]$$

Eqn (2.4) is valid for $0 \le i \le n-1$, otherwise

$$A^n_{n,n} = \binom{n}{n}\binom{n}{n} n! = n!$$

D. NUMERICAL CALCULATIONS AND DISCUSSIONS

For the MSCRA protocol two major computed programs were written. The first one provides numerical values for $\bar{L}_n$ of Eqn (2.3). The other program gives simulation results to verify $\bar{L}_n$.

1. Numerical Calculations of $\bar{L}_n$

The first program was written in WATFIV using double precision and real value variable declarations. This program recursively generates the $A^n{}_{n,i}$ of Eqn (2.4) in the following order;

$$
\begin{array}{llll}
A_{0,0} & & & \\
A_{1,1} & A_{1,0} & & \\
A_{2,2} & A_{2,1} & A_{2,0} & \\
A_{3,3} & A_{3,2} & A_{3,1} & A_{3,0} \quad \cdots
\end{array}
$$

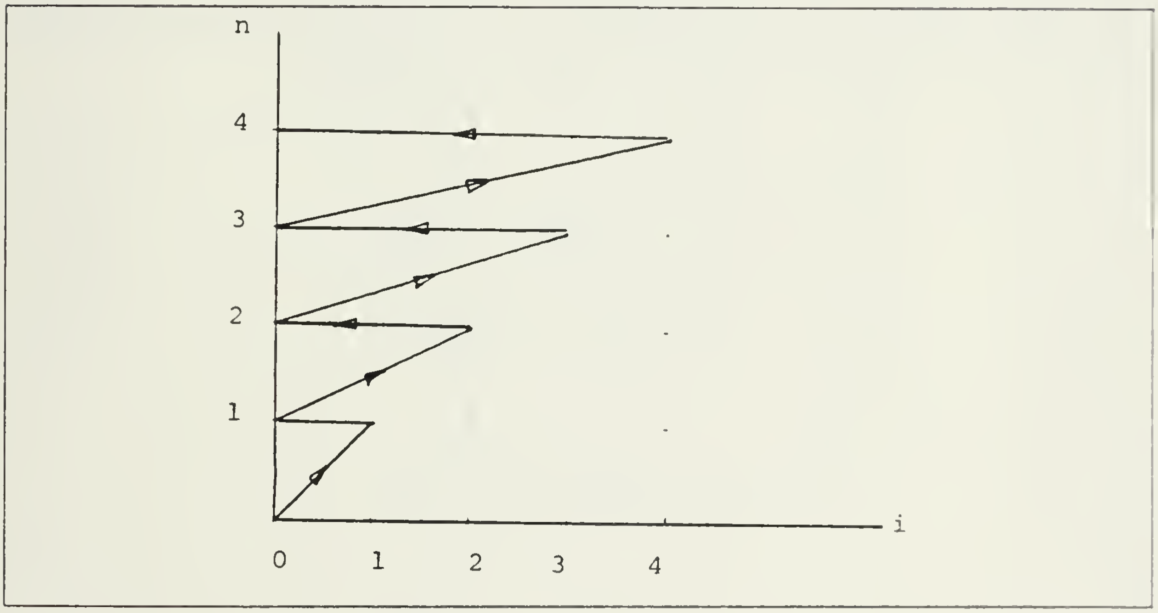

Figure 2.3 The Relationship Between n and i

As demonstrated in Figure 2.3, i cannot be greater than n (the number of successes cannot be greater than the number of users). After calculation of the $A^n{}_{n,i}$, we can obtain $\bar{L}_n$. easily. A function subprogram was written to calculate the FACTORIAL. The Watfiv program is given in Appendix A.

2. Simulation

The second program gives simulation results by using GGUD (a random number generator) from the IMSL routine library. GGUD is used to control the random retransmission of the packets. The flow diagram of the simulation is illustrated in Figure 2.4 . A program listing is provided in Appendix B.

3. Examples and Discussions

Table I gives a comparison between the $\bar{L}_n$ obtained by analytical formula Eqn (2.3) and that obtain by simulation. We observe that the agreement between analysis and simulation is extremely good. Since the error is always less than 1%, thus verifies the correctness of the analysis done for the MSCRA. This comparison is also represented graphically in Figure 2.5.

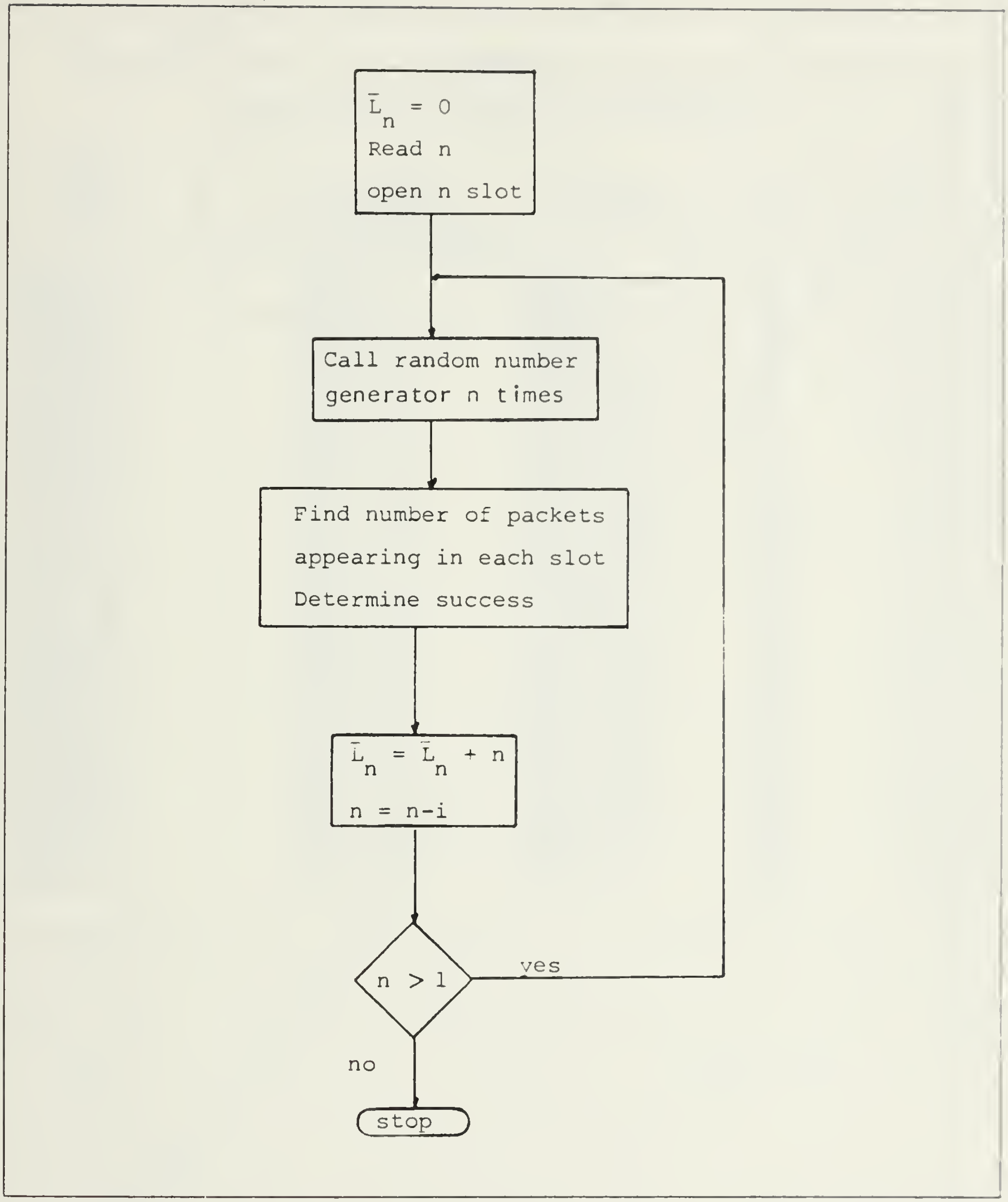
$\bar{L}_n = 0$
Read n
open n slot
Call random number generator n times
Find number of packets appearing in each slot
Determine success
$\bar{L}_n = \bar{L}_n + n$
n = n-i
n > 1
yes
no
stop

Figure 2.4 Simulation Path

TABLE I

SIMULATION AND ANALYSIS FOR MSCRA

N	B	ANALYSIS	SIMULATION	ERROR %
1	1	0.0000	0.0000	0.0000
2	2	4.0000	4.0338	-0.8450
3	3	6.3750	6.4121	-0.5820
4	4	8.8240	8.8565	-0.3683
5	5	11.3260	11.3625	-0.3223
6	6	13.8650	13.9101	-0.3253
7	7	16.4290	16.4825	-0.3256
8	8	19.0120	18.9938	0.0957
9	9	21.6100	21.5463	0.2948
10	10	24.2200	24.2977	-0.3208
11	11	26.8390	26.9795	-0.5235
12	12	29.4670	29.4177	0.1673
13	13	32.1020	32.0365	0.2040
14	14	34.7430	34.8406	-0.2809
15	15	37.3890	37.3360	0.1418
16	16	40.0390	40.0077	0.0782
17	17	42.6930	42.6357	0.1342
18	18	45.3510	45.2560	0.2095
19	19	48.0120	48.0679	-0.1164
20	20	50.6760	50.6743	0.0034
21	21	53.3430	53.2771	0.1235
22	22	56.0110	55.9586	0.0936
23	23	58.6820	58.7231	-0.0700
24	24	61.3550	61.3038	0.0834
25	25	64.0300	64.1019	-0.1123
26	26	66.7070	66.7245	-0.0262
27	27	69.3850	69.2884	0.1392
28	28	72.0640	72.0177	0.0642
29	29	74.7450	74.7899	-0.0601
30	30	77.4270	77.5541	-0.1642

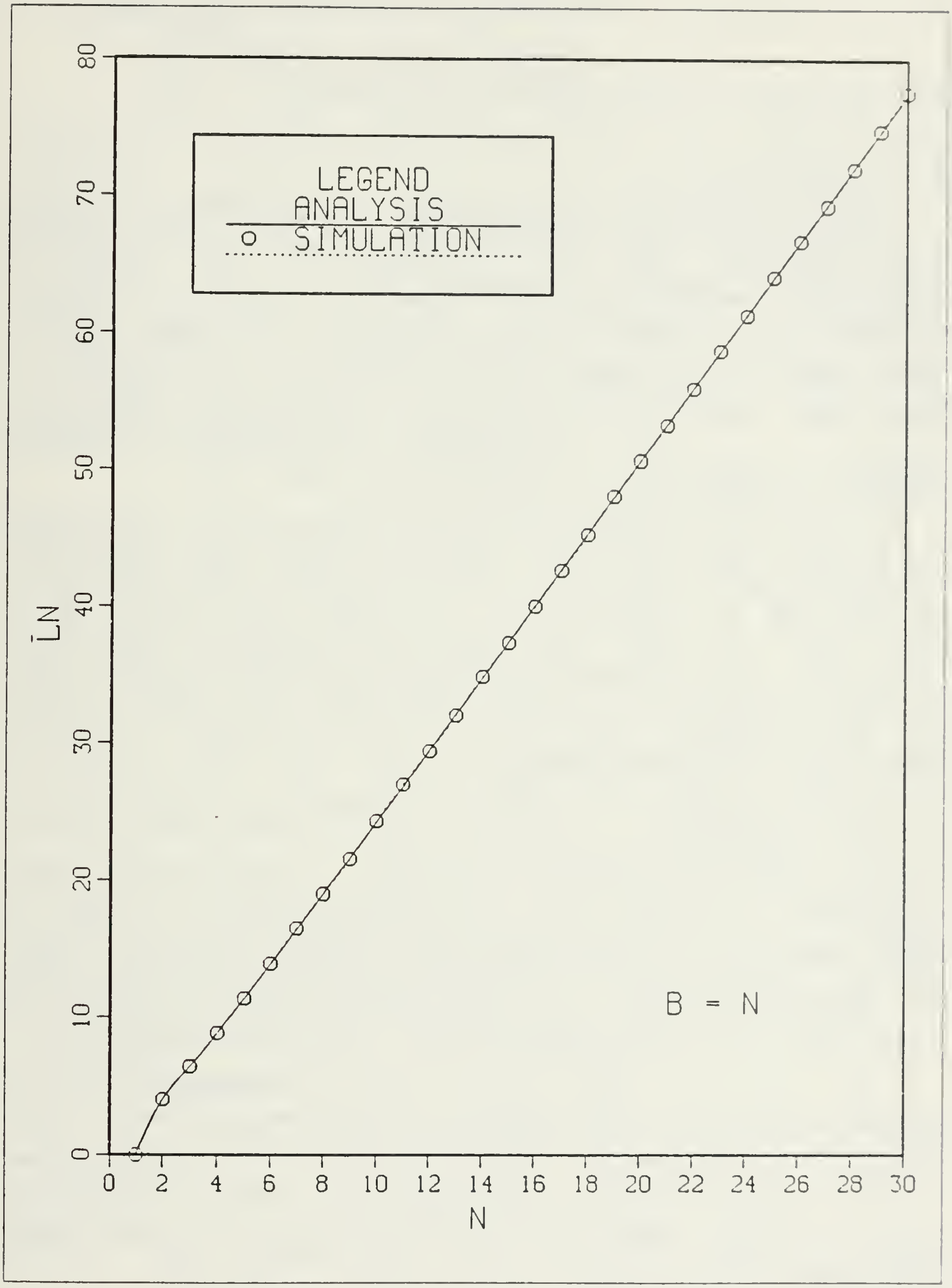

Figure 2.5 Simulation and Analysis for MSCRA

III. ADAPTIVE MULTISLOT CRA

A. INTRODUCTION

The efficiency of the MSCRA can be improved by modifying the algorithm such that counting for the number of successful transmissions can be made in an adaptive manner. As described in the previous chapter, the MSCRA opens n slots when a collision involving n users is detected. It then waits until the end of the n slot period, counts the number of successful transmissions and restarts with the remaining packets yet to be resolved.

In our new algorithm, upon the detection of a collision involving n users we still open n slots but make observation on only the the first b ($b \leq n$) slots. At the end of the bth slot, we count the number of successes which have occured in the first b slots and repeat the process. This new algorithm will be called Adaptive Multislot Collision Resolution Algorithm (AMSCRA).

In this chapter, we shall consider the following two types of AMSCRA:

(1) If i successes have occured in the first b slots, repeat the collision resolution procedure with n-i users. In other words, without waiting until the end of the nth slot, we immediately open n-i slots and proceed similarly. We shall call this policy AMSCRA with collective resolution.

(2) If j users have tried their retransmissions in the first b slots and among them i have succeeded, then repeat the process immediately with j-i users. After each of these j-i users have successfully transmitted, restart the process again with n-j users. This policy will be called AMSCRA with separate resolution.

B. AMSCRA WITH COLLECTIVE RESOLUTION

1. Description of the Protocol

After a collision involving n users is detected, the system opens n slots. Each of colliding users randomly picks a slot and retransmits. Users participating in the collision resolution observe the status of the channel for the first b slots. Let i represent the number of successful packets in the first b slots. This process is repeated at the end of the bth slot with n-i users. If the number of the packets to be resolved n, becomes smaller than b, then open n slots and make observations over this n-slot period. This case reduces to the MSCRA of Chapter II.

In the example of figure 3.1 the energy detector indicates that the number of packets in the first slot is 5. The resolution algorithm opens five slots and observes the first two slots (slots 2 and 3) because b=2. Unfortunately, no successes are observed in slots 2 and 3 so system opens another five slots following slot 3 and checks 2 slots again (slots 4 and 5). Now we see a success for message A in slot 4. However there are still four collided packets which need to be resolved. The systems opens four slots, checks slots 6 and 7 and finds two more successes for messages B and D in this third interval. The total number of successful packets is now 3. The fourth interval is now opened and messages C and E are also observed to be successful. Thus the entire collision resolution interval(CRI) is completed. In this particular example, the CRI requires a total of 9 slots.

2. Analysis

Before proceeding to derive $\bar{L}_n$ a few definitions are in order.

$V^b_{j,i}$ = The probability that out of n users j of them retransmit in the b slots and result in i succees.

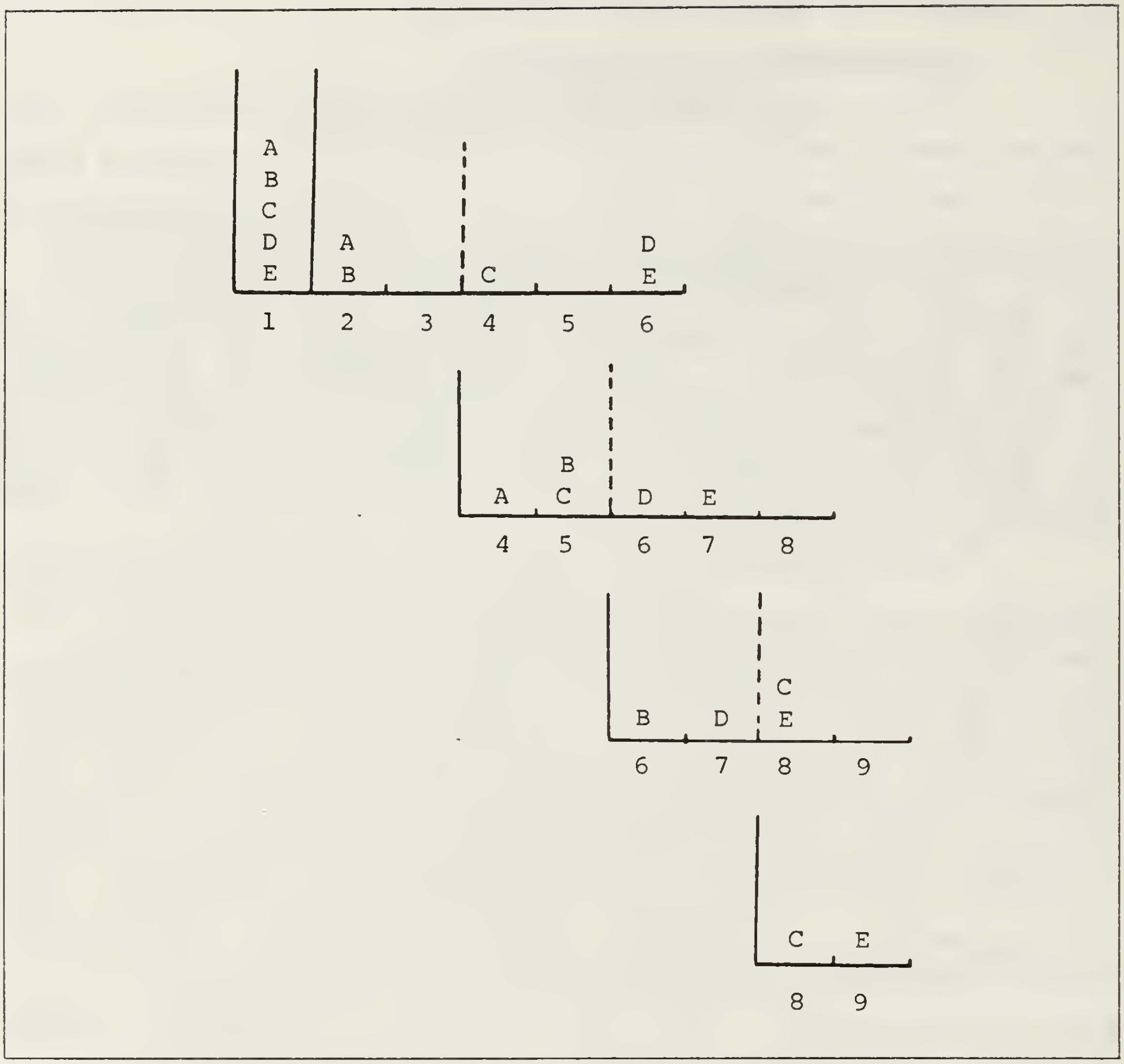

Figure 3.1 An Example for AMSCRA with Collective Resolution b=2

$V^b_{j,i}$ = Prob[i successes/j users retransmit in the 1st. b slots]. Prob[Among n users j of them retransmit in the 1st.b slots]

$A^b_{j,i}$ = The number of the arrangements in distributing j packets into b slots, which result in i successes.

In general we can write,

$$A^{b}_{j,i} = \binom{b}{i}\binom{j}{i} i! \left[(b-i)^{j-i} - A^{b-i}_{j-i,1} - A^{b-i}_{j-i,2} \cdots - A^{b-i}_{j-i,j-i}\right] \qquad (3.2)$$

Eqn (3.2) is expressed in recursive form and is valid for $0 \le i \le \min(b,j)-1$ The factor $\binom{b}{i}$ represents the number of arrangments in selecting i packets from b slots and $\binom{j}{i}$ represents the number of arrangments possible in selecting i packets out of j packets. The factor i! represents the number of arrangments possible in distributing i packets into i slots such that each slot is a success. In Eqn (3.2), (b-i) represents the number of arrangments in distributing the remaining j-i packets into b-i slots which result in no successes. If i=min(b,j), then,

$k=\min(b,j)$

$$A_{j,k}^{b} = \binom{b}{k}\binom{j}{k} k! \cdot U(b-j+1) \qquad (3.3)$$

$$\text{where} \quad U(x) = \begin{cases} 1 & x>0 \\ 0 & \text{otherwise} \end{cases}$$

The U(x) in Eqn (3.2) is used to prevent $b \ge i$, $i=\min(b,j)$ from occurring. Based on the definitions of $V^{b}_{j,i}$ and $A^{b}_{j,i}$, we have

$$V_j{}^b{}_i = \frac{A_{j.i}^{b}}{b^j}\binom{n}{j}\left[\frac{b}{n}\right]^j \left[1 - \frac{b}{n}\right]^{n-j} \qquad (3.4)$$

With this probability the system state proceeds from n to n-i, i.e., the situation illustrated in Figure 3.2.

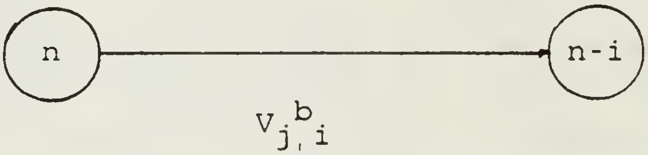

Figure 3.2 Transition From State n to n-i if $n \geq b$

Thus if $n \geq b$.

$$\bar{L}_n = b + \sum_{j=0}^{n} \sum_{i=0}^{\min(b,j)} V_{j,i}^{b} \left(\hat{L}_{j-i} + \hat{L}_{n-j} \right) \tag{3.5}$$

$$\text{where } \hat{L}_n = \begin{cases} 1 & n=1 \\ \bar{L}_n & \text{otherwise} \end{cases}$$

The relationship between $\bar{L}_n$ and $\hat{L}_n$ is necessary because if $n-i = 1$, then we need one more slot to complete the CRI. However L_1 has been previously defined to be 0. Finally, the actual number of time slots needed in successful transmitting n packets is

$$\bar{L}^{d}_{n} = \bar{L}_n + 1$$

If n < b as we pointed out earlier in this chapter, the result reduces to that in Chapter II, thus

$$\bar{L}_n = \begin{cases} \dfrac{1}{1-P_{n,0}} \quad n + \displaystyle\sum_{i=1}^{n} P_{n,i}\, \hat{L}_{n-i} & n \le b \\[2em] \dfrac{1}{1-\displaystyle\sum_{j=0}^{n} V^{n}_{j,0}} \quad b + \displaystyle\sum_{i=0}^{\min(b,j)} V^{b}_{j,i}\, \hat{L}_{n-i} & n > b \end{cases} \qquad (3.6)$$

Where

$$\hat{L}_n = \begin{cases} 1 & n=1 \\ \bar{L}_n & \text{otherwise} \end{cases}$$

3. Numerical Calculations and Discussions

a. Numerical Calculation of $\bar{L}_n$

A computer program to calculate $\bar{L}_n$ was written in WATFIV. Double precision and real value variable declarations were used. This program creates $A^{b}_{j,i}$ of Eqn (3.2) and Eqn (3.3) recursively. This program differs from the program of Chapter 2 in that we must ensure that i is never greater than b or j (the number of successes is limited by the number of slots and the number of users). The Watfiv program of AMSCRA with collective resolution is given in Appendix A.

b. Simulation

A simulation program was written in FORTRAN. GGUD was used as a random number generator. The Fortran program is provided in Appendix B. A flow diagram of the simulation is illustrated in Figure 3.3.

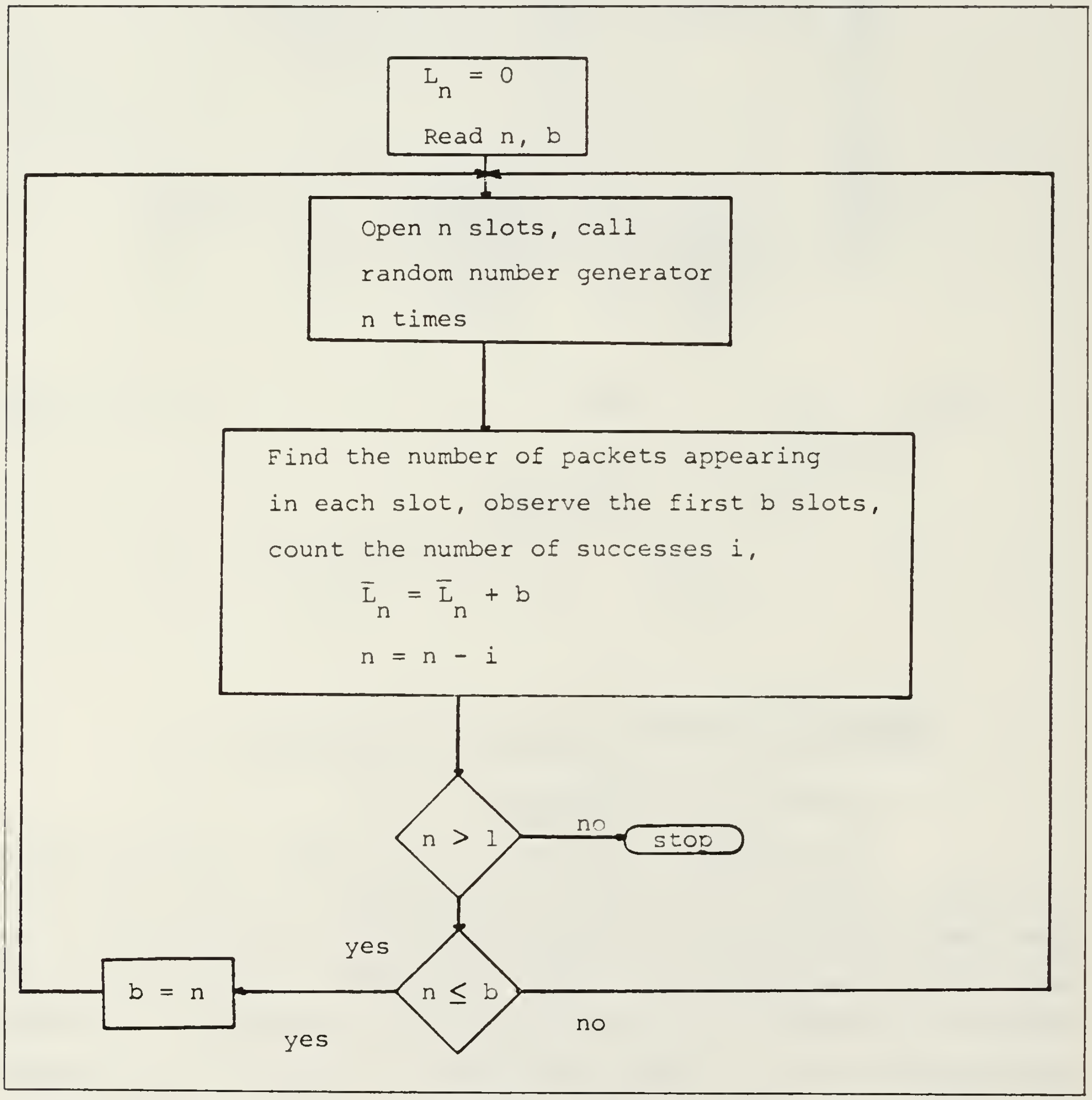
$L_n = 0$
Read n, b
Open n slots, call random number generator n times
Find the number of packets appearing in each slot, observe the first b slots, count the number of successes i,
$\bar{L}_n = \bar{L}_n + b$
$n = n - i$
$n > 1$
no
stop
yes
$n \leq b$
no
yes
$b = n$

Figure 3.3 Simulation Flow Diagram for AMSCRA with Collective Resolution

c. Examples and Discussion

Table II on p. 47 gives a comparison between the $\bar{L}_n$ obtained by analytical formula and those obtained by simulation. We observe that the agreement between analysis and simulation is extremely close. This verifies the correctness of analysis done for this protocol. Comparison is also done graphically in Figures 3.4, 3.5, 3.6 and 3.7. From Figure 3.4 we observe that b = 1 offers the best performance. Comparison between MSCRA and AMSCRA will be made at the end of this chapter.

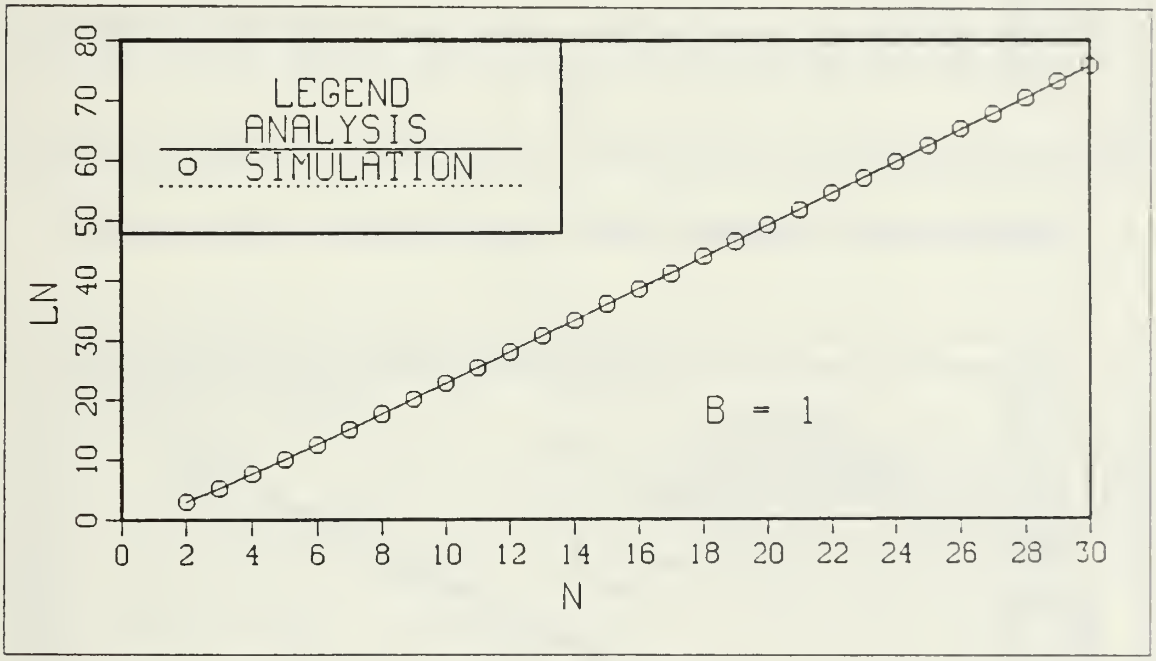

Figure 3.4 AMSCRA with Collective Resolution b=1

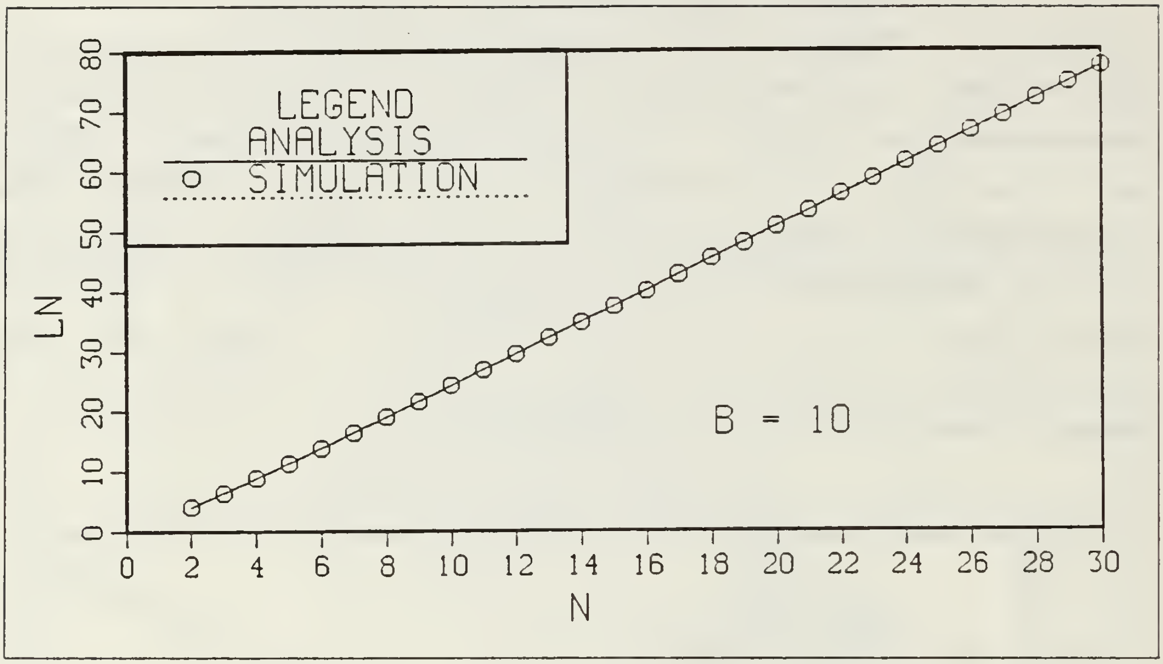

Figure 3.5 AMSCRA with Collective Resolution b=10

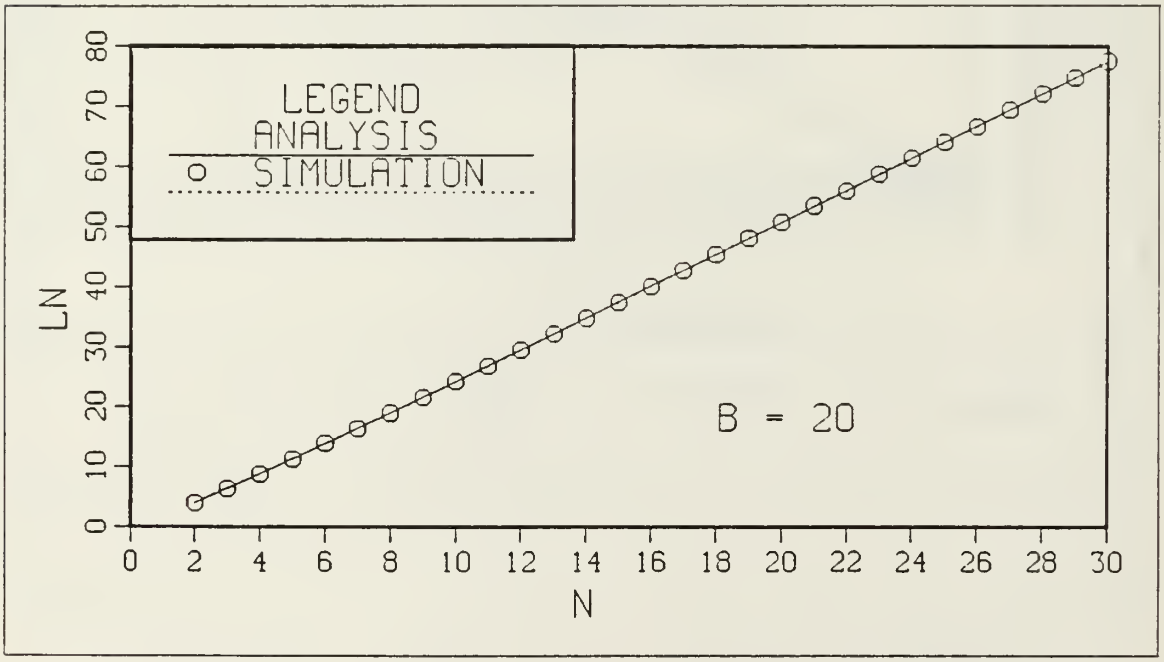

Figure 3.6 AMSCRA with Collective Resolution b=20

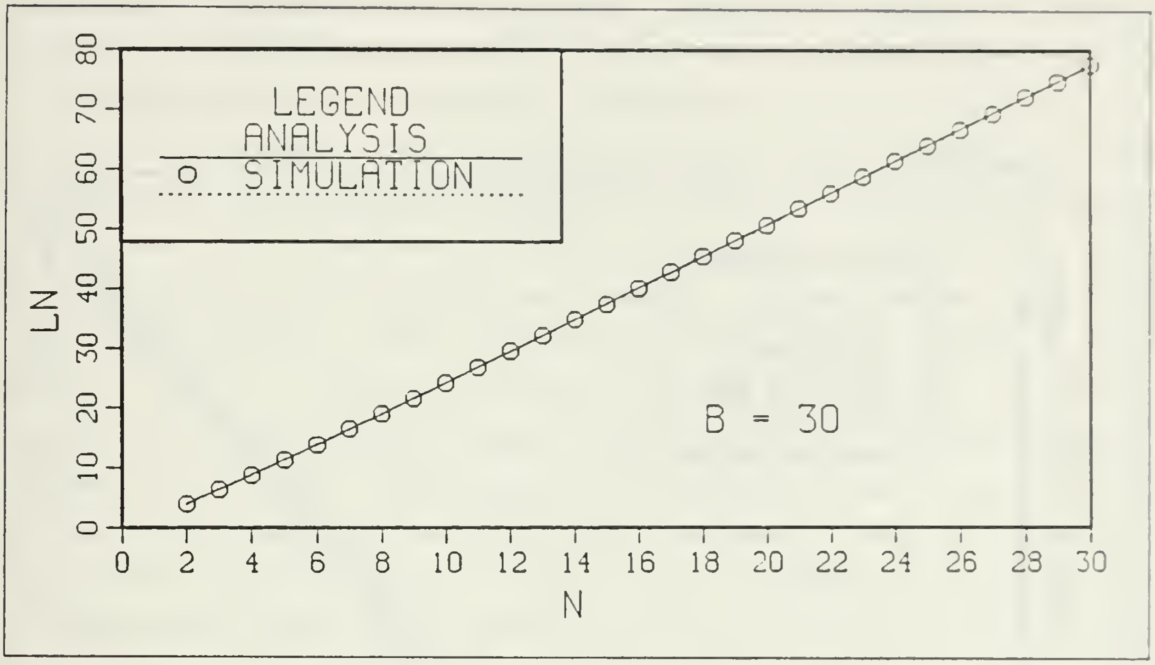

Figure 3.7 AMSCRA with Collective Resolution b=30

C. AMSCRA WITH SEPARATE RESOLUTION

1. Description

After a collision involving n users is detected the systems opens n slots. Again each user randomly picks one slot and retransmits. All the participating users observe the channel until the end of the first b slots j out of n users have tried in the first b slots resulting in i successes. Then we repeat the process twice with j-i users and the other with n-j users. In other words, at the end of the bth slot we immediately open j-i slots to resolve the collision among the remaining j-i users. After each of these j-i users has successfully transmitted. We open n-j slots to resolve the collision among the remaining n-j users. In any case, if n < b then we only open n slots and make observation for these n slots.

In Figure 3.9, the collision in slot 1 involves five users. The system therefore opens five slots for collision resolution. Each of these five collided users selects one

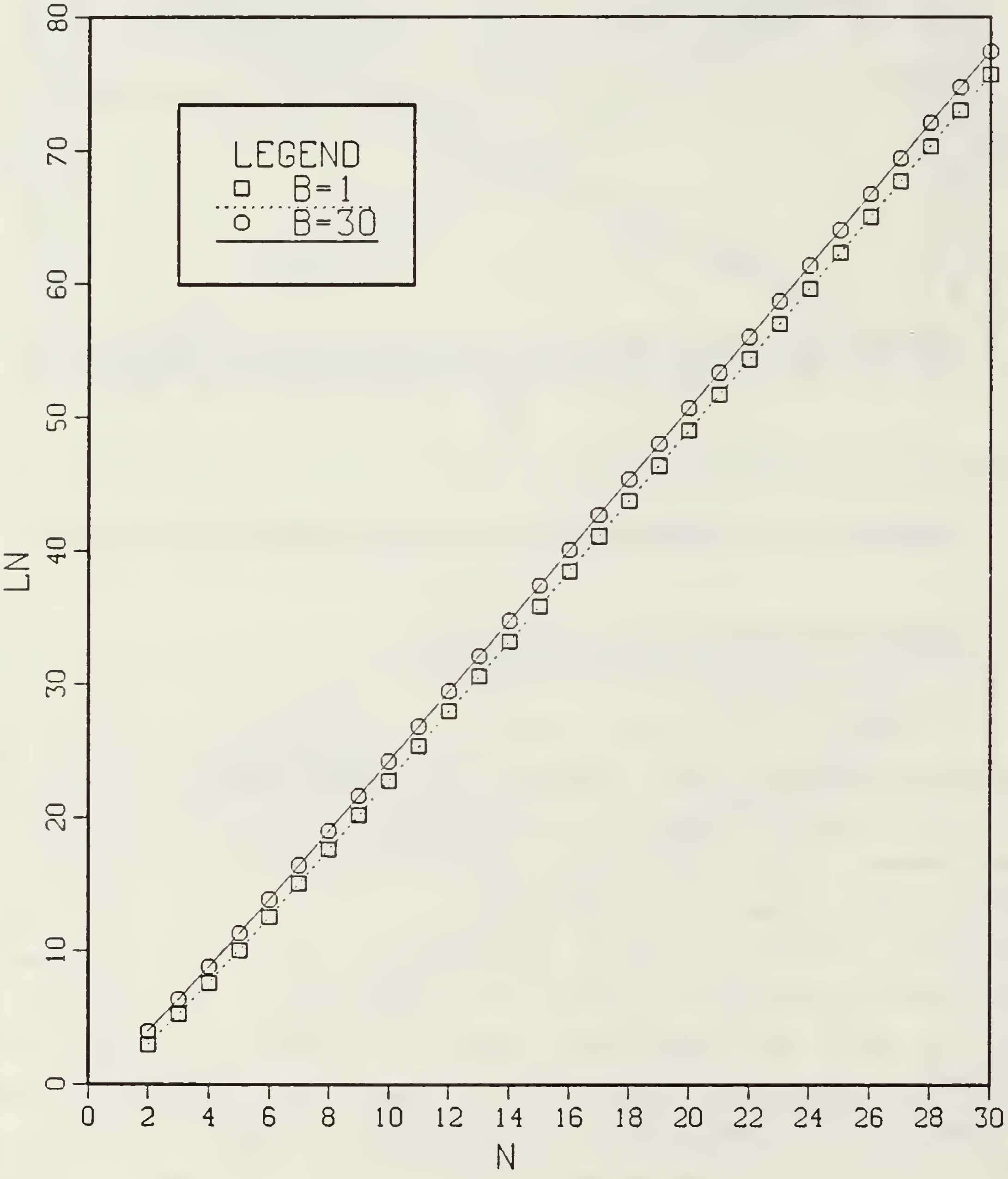

Figure 3.8 AMSCRA with Collective Resolution

slot for retransmission randomly. In this example A and B select slot 2, D and E select slot 6 and user C selects slot 4. We divide the collision resolution into two parts. the first part covers the first b slots (slot 2 and 3), the second part covers the remaining slots. The system stores the users which appear in the second part (user C in slot 4). In the first b slots we observe no successes and four collided packets (j=4); therefore the system opens 4 more slots, (4, 5, 6 and 7). There are now two users, A and B, involved in a collision the first two slots of the 4 newly opened slots. The algorithm then opens two more slots (slots 6 and 7) to resolve the collision between A and B. Users A and B now successfully transmit their packets in slots 6 and 7. After this step, the system opens two slots (8 and 9) for the previous stored packets D and E. Both D and E now succeed. Finally the system opens one more slot for packet C.

2. Analysis

After the observation of the first b slots, the group of n collided users will be subdivided into two separate groups with sizes j-i and n-j, j represents the number of users which retransmited in the first b slots. This situation is now demonstrated in Figure 3.10 . $V^b_{j,i}$ has been defined in Section III.B.2.

From figure 3.10, if b ≥ n we have,

$$\bar{L}_n = b + \sum_{j=0}^{n} \sum_{i=0}^{\min(b,j)} V^b_{j,i}(\hat{L}_{j-i} + \hat{L}_{n-i}) \qquad (3.7)$$

$$\text{where} \quad \hat{L}_n = \begin{cases} 1 & \text{if } n=1 \\ \bar{L}_n & \text{otherwise} \end{cases}$$

and
$$\bar{L}_0 = \bar{L}_1 = 0$$
$$\bar{L}^d_n = \bar{L}_n + 1$$

In general

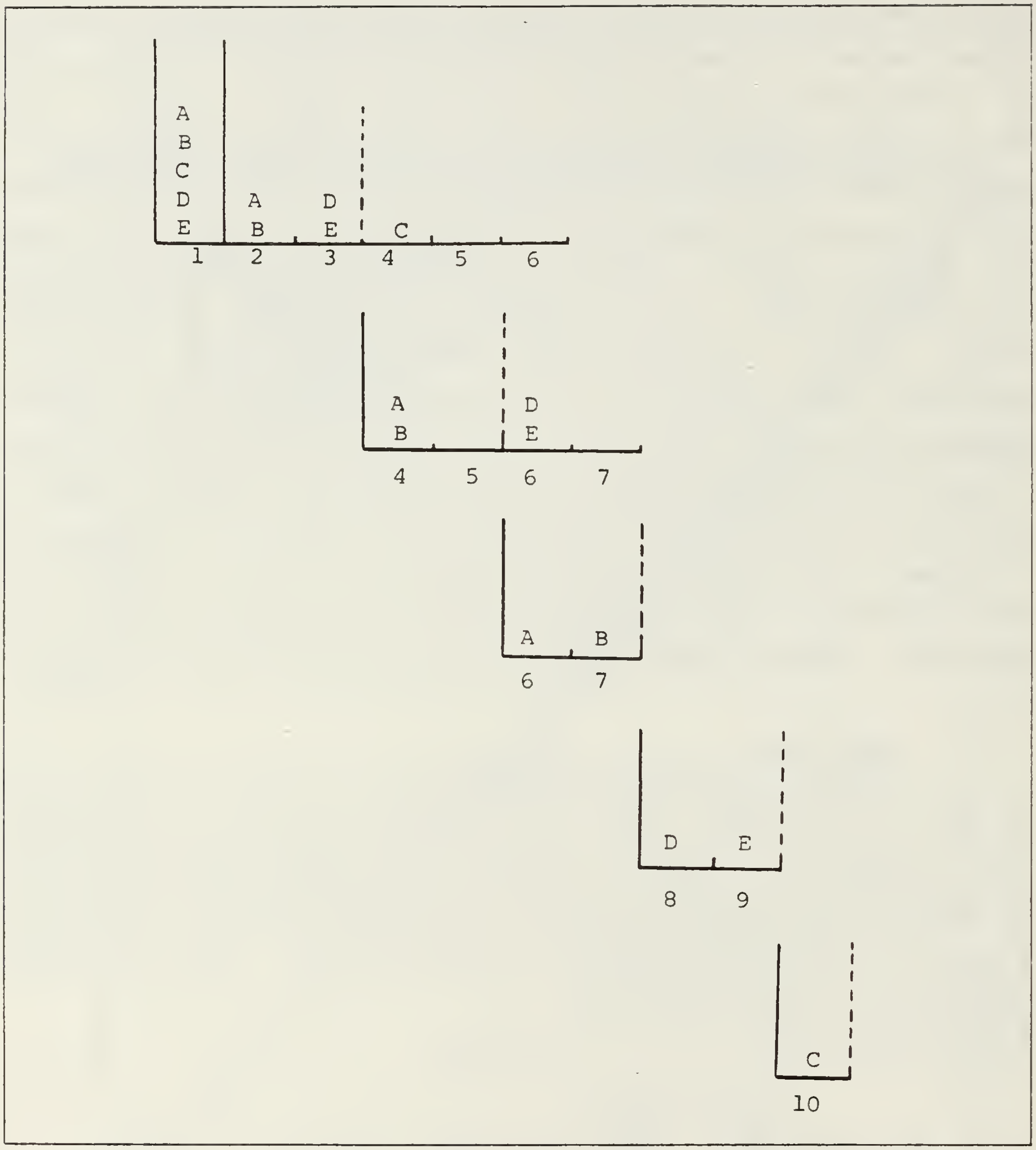

Figure 3.9 AMSCRA with Separated Resolution
b=2

$$\bar{L}_n = \begin{cases} n + \displaystyle\sum_{i=0}^{n} P_{n,i}\hat{L}_{n-i} & n \le b \\ b + \displaystyle\sum_{j=0}^{n} \sum_{i=0}^{\min(b,j)} V^b{}_{j,i}\,(\hat{L}_{j-i} + \hat{L}_{n-j}) & n > b \end{cases}$$

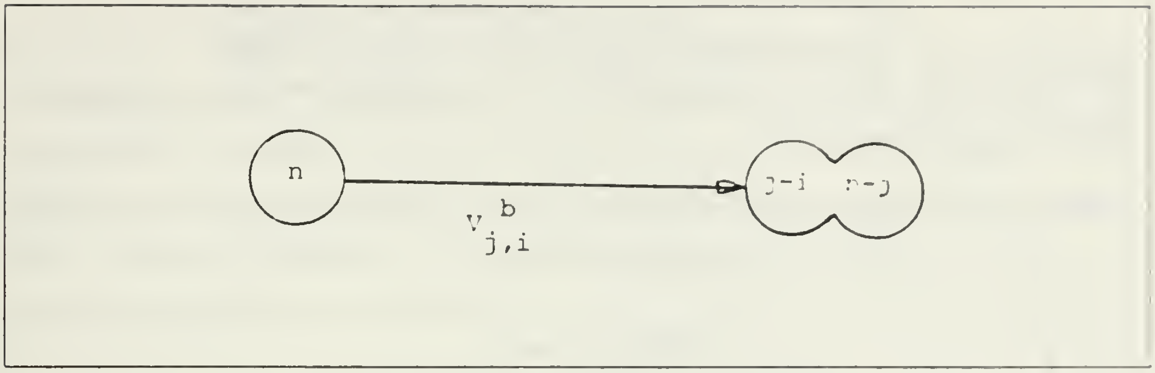

Figure 3.10 State Transition Diagram for AMSCRA with Separated Resolution

$$\bar{L}_n = \begin{cases} \dfrac{1}{1-P_{n,0}} \; n + \displaystyle\sum_{i=0}^{n} P_{n,i}\,\hat{L}_{n-i} & n \le b \\[2ex] \dfrac{1}{1-V^n_{n,0} - V^n_{0,0}} \; b + \displaystyle\sum_{i=1}^{\min(b,j)} V^b_{n,i}\,\hat{L}_{n-i} + \sum_{j=1}^{n-1}\sum_{i=0}^{\min(b,j)} V^b_{j,i}\,(\hat{L}_{j-i} + \hat{L}_{n-j}) & \end{cases} \tag{3.8}$$

3. Numerical Calculations and Discussions

a. Numerical Calculation of $\bar{L}_n$

The program for calculating $\bar{L}_n$ is a quite similar to that used for AMSCRA with collective resolution. A Watfiv program of the AMSCRA with separate collision resolution is given in Appendix C.

b. Simulation

The simulation program was written in Fortran and is provided in Appendix D. A simulation flow diagram is shown in Figure 3.11 .

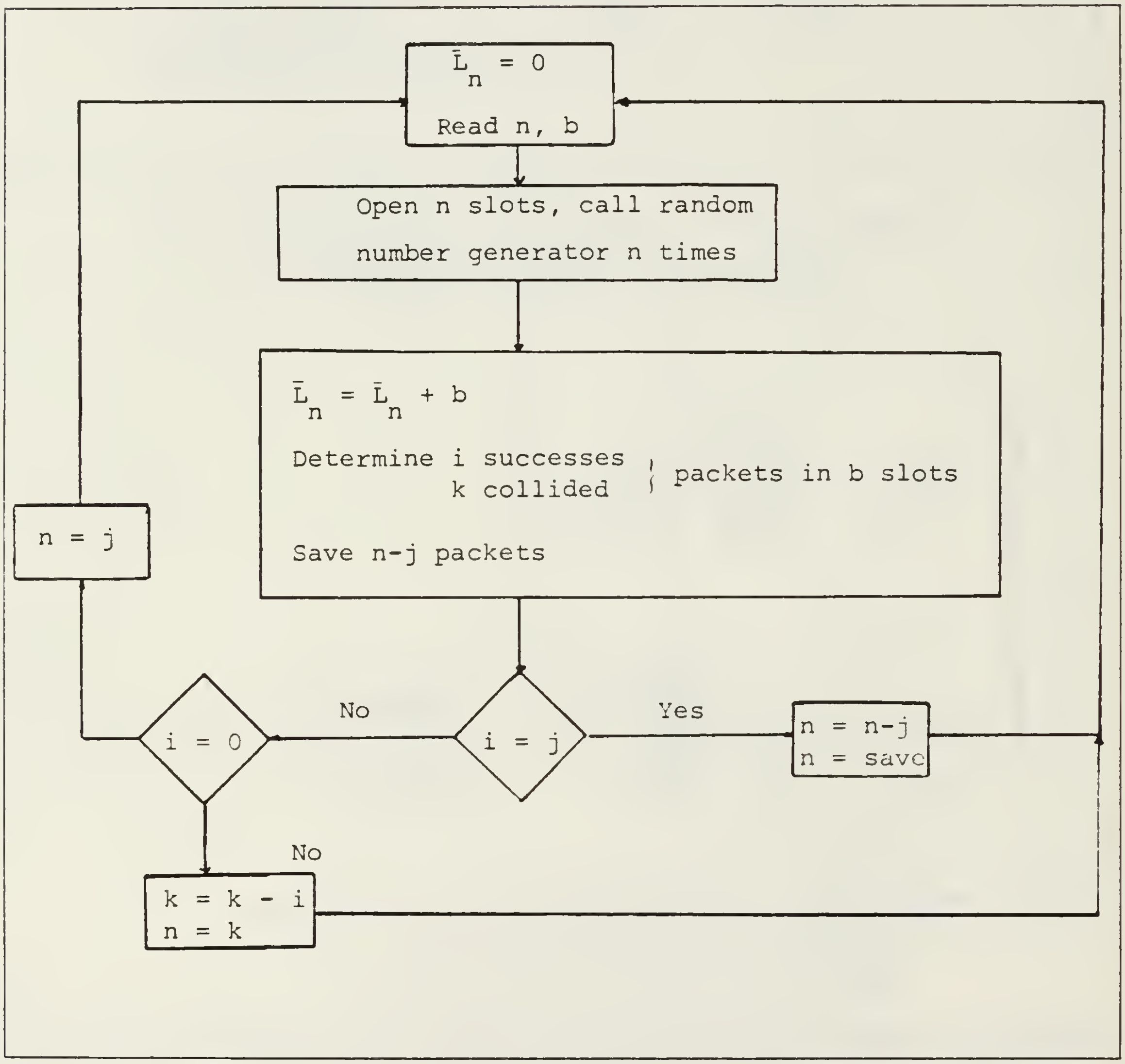
$\bar{L}_n = 0$
Read n, b
Open n slots, call random number generator n times
$\bar{L}_n = \bar{L}_n + b$
Determine i successes, k collided packets in b slots
Save n-j packets
n = j
i = 0
No
i = j
Yes
n = n-j
n = save
No
k = k - i
n = k

Figure 3.11 Simulation Flow Diagram for AMSCRA with Separate Resolution.

c. Examples and Discussions

The comparison between simulation and analysis of AMSCRA with separated resolution is given in Table II. Agreement between analysis and simulation is good. This verifies the analysis of the AMSCRA with separated resolution. Some graphical results are given in Figure 3.12.

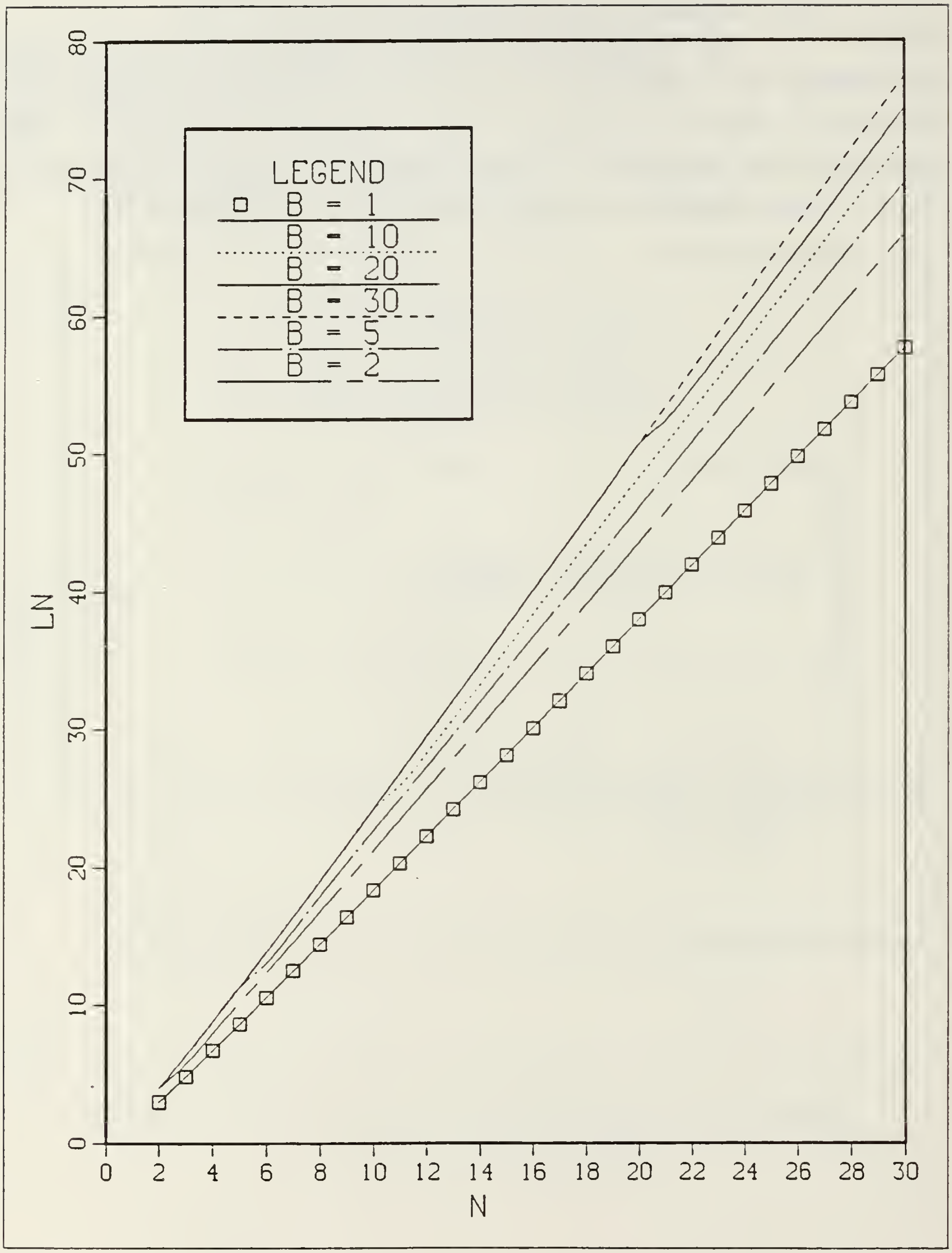

Figure 3.12 AMSCRA with Separate Resolution

TABLE II

ANALYSIS AND SIMULATION RESULTS FOR TWO VERSIONS OF AMSCRA

		AMSCRA WITH COL.RESOL.			AMSCRA WITH SEP.RESOL.		
n	b	analy.	sim.	error %	analy.	sim.	error %
2	1	3.0000	2.9875	0.42	3.0000	2.9875	0.42
3	1	5.2500	5.2373	0.24	4.8330	4.8362	-0.07
4	1	7.6200	7.6553	-0.46	6.7350	6.7389	-0.06
5	1	10.0610	10.0767	-0.16	8.6510	8.6737	-0.26
6	1	12.5500	12.5165	0.27	10.5780	10.5717	0.06
7	1	15.0710	15.0786	-0.05	12.5130	12.5203	-0.06
8	1	17.6180	17.6317	-0.08	14.4550	14.4446	0.07
9	1	20.1840	20.1598	0.12	16.4010	16.3929	0.05
10	1	22.7650	22.8283	-0.28	18.3510	18.3012	0.27
11	1	25.3580	25.3789	-0.08	20.3040	20.3043	0.00
12	1	27.9630	27.9407	0.08	22.2600	22.3000	-0.18
13	1	30.5760	30.6459	-0.23	24.2180	24.2087	0.04
14	1	33.1960	33.1607	0.11	26.1770	26.2009	-0.09
15	1	35.8230	35.9512	-0.36	28.1390	28.1075	0.11
16	1	38.4560	38.4183	0.10	30.1020	30.1158	-0.05
17	1	41.0940	41.0271	0.16	32.0660	32.0715	-0.02
18	1	43.7360	43.8518	-0.26	34.0310	33.9959	0.10
19	1	46.3830	46.3764	0.01	35.9980	36.0375	-0.11
20	1	49.0330	49.1045	-0.15	37.9650	37.9807	-0.04
21	1	51.6860	51.6110	0.15	39.9330	39.9587	-0.06
22	1	54.3420	54.4449	-0.19	41.9010	41.8924	0.02
23	1	57.0010	56.9377	0.11	43.8710	43.9024	-0.07
24	1	59.6630	59.6553	0.01	45.8410	45.8443	-0.01
25	1	62.3260	62.3779	-0.08	47.8120	47.7657	0.10
26	1	64.9920	65.1219	-0.20	49.7830	49.7609	0.04
27	1	67.6600	67.6361	0.04	51.7540	51.7794	-0.05
28	1	70.3290	70.3214	0.01	53.7260	53.7503	-0.05
29	1	73.0010	73.1116	-0.15	55.6990	55.6441	0.10
30	1	75.6730	75.6847	-0.02	57.6720	57.6199	0.09
2	2	4.0000	4.0338	-0.85	4.0000	3.9981	0.05
3	2	6.0000	6.0191	-0.32	5.7500	5.7520	-0.03
4	2	8.4760	8.4859	-0.12	7.9740	7.9615	0.16
5	2	10.9080	10.9185	-0.10	10.1820	10.1225	0.58
6	2	13.4110	13.4549	-0.33	12.3700	12.3175	0.42
7	2	15.9370	16.0091	-0.45	14.5730	14.5907	-0.12
8	2	18.4880	18.5214	-0.18	16.7850	16.8049	-0.12
9	2	21.0580	21.0571	0.00	19.0000	19.0123	-0.06
10	2	23.6420	23.6123	0.13	21.2200	21.1698	0.24
11	2	26.2380	26.3397	-0.39	23.4430	23.4181	0.11
12	2	28.8440	28.9102	-0.23	25.6680	25.6465	0.08
13	2	31.4590	31.3674	0.29	27.8970	27.9193	-0.08
14	2	34.0810	34.0205	0.18	30.1270	30.2095	-0.27
15	2	36.7090	36.6229	0.23	32.3590	32.4653	-0.33
16	2	39.3430	39.3455	-0.01	34.5920	34.5955	-0.01
17	2	41.9820	41.8993	0.20	36.8270	36.8322	-0.01
18	2	44.6250	44.6759	-0.11	39.0630	39.0846	-0.06
19	2	47.2720	47.3630	-0.19	41.3000	42.0100	-1.72
20	2	49.9230	49.8597	0.13	43.5390	44.3087	-1.77
21	2	52.5770	52.4804	0.18	45.7780	45.7496	0.06
22	2	55.2330	55.1759	0.10	48.0180	47.9881	0.06
23	2	57.8930	58.0323	-0.24	50.2580	50.3245	-0.13
24	2	60.5550	60.5627	-0.01	52.5000	52.3339	0.32
25	2	63.2190	63.0178	0.32	54.7420	54.7832	-0.08
26	2	65.8850	65.8023	0.13	56.9850	57.0878	-0.18
27	2	68.5530	68.4571	0.14	59.2280	59.2375	-0.02

		AMSCRA WITH COL.RESOL.			AMSCRA WITH SEP.RESOL.		
n	b	analy.	sim.	error %	analy.	sim.	error %
28	2	71.2230	71.2687	-0.06	61.4720	61.4021	0.11
29	2	73.8940	73.9437	-0.07	63.7160	63.8025	-0.14
30	2	76.5680	76.5864	-0.02	65.9600	66.0833	-0.19
2	3	4.0000	3.9534	1.16	4.0000	3.9534	1.16
3	3	6.3750	6.4121	-0.58	6.3750	6.3940	-0.30
4	3	8.6690	8.6306	0.44	8.1280	8.1353	-0.09
5	3	11.1850	11.2066	-0.19	10.4570	10.4696	-0.12
6	3	13.6780	13.6823	-0.03	12.7780	12.7816	-0.03
7	3	16.2130	16.2292	-0.10	15.0110	14.9729	0.25
8	3	18.7690	18.7421	0.14	17.2480	17.2083	0.23
9	3	21.3410	21.3636	-0.11	19.5090	19.5243	-0.08
10	3	23.9280	23.9389	-0.05	21.7770	21.8382	-0.28
11	3	26.5260	26.4859	0.15	24.0450	24.0835	-0.16
12	3	29.1340	29.0908	0.15	26.3150	26.2545	0.23
13	3	31.7510	31.7960	-0.14	28.5870	28.5815	0.02
14	3	34.3740	34.4127	-0.11	30.8620	30.7996	0.20
15	3	37.0030	36.9507	0.14	33.1390	33.1091	0.09
16	3	39.6380	39.6465	-0.02	35.4180	35.4469	-0.08
17	3	42.2780	42.1911	0.21	37.6970	37.7141	-0.05
18	3	44.9220	44.8355	0.19	39.9790	39.9558	0.06
19	3	47.5700	47.4865	0.18	42.2610	42.7746	-1.22
20	3	50.2210	50.1943	0.05	44.5440	45.2575	-1.60
21	3	52.8760	52.9734	-0.18	46.8290	46.8515	-0.05
22	3	55.5330	55.5329	0.00	49.1140	48.9929	0.25
23	3	58.1930	58.1899	0.01	51.4000	51.3803	0.04
24	3	60.8550	60.7276	0.21	53.6870	53.7095	-0.04
25	3	63.5200	63.5012	0.03	55.9740	56.0682	-0.17
26	3	66.1870	66.1415	0.07	58.2630	58.2945	-0.05
27	3	68.8550	68.6945	0.23	60.5510	60.3636	0.31
28	3	71.5250	71.4855	0.06	62.8410	62.9126	-0.11
29	3	74.1970	74.1577	0.05	65.1300	65.0876	0.07
30	3	76.8700	76.7085	0.21	67.4210	67.4597	-0.06
2	4	4.0000	3.9962	0.10	4.0000	3.9962	0.10
3	4	6.3750	6.3585	0.26	6.3750	6.3585	0.26
4	4	8.8240	8.8565	-0.37	8.8240	8.8007	0.26
5	4	11.2500	11.3325	-0.73	10.5870	10.5807	0.06
6	4	13.7880	13.8047	-0.12	12.9460	12.9342	0.09
7	4	16.3220	16.2813	0.25	15.3710	15.4151	-0.29
8	4	18.8810	18.7971	0.44	17.6940	17.6950	-0.01
9	4	21.4580	21.3435	0.53	19.9600	19.8617	0.49
10	4	24.0470	24.0914	-0.18	22.2420	22.2353	0.03
11	4	26.6480	26.6587	-0.04	24.5550	24.5163	0.16
12	4	29.2580	29.2451	0.04	26.8760	26.9021	-0.10
13	4	31.8760	31.9949	-0.37	29.1930	29.2893	-0.33
14	4	34.5000	34.5113	-0.03	31.5080	31.5212	-0.04
15	4	37.1310	37.1046	0.07	33.8240	33.8729	-0.14
16	4	39.7670	39.8325	-0.16	36.1440	36.0943	0.14
17	4	42.4080	42.3793	0.07	38.4650	38.3898	0.20
18	4	45.0530	45.0248	0.06	40.7880	40.7629	0.06
19	4	47.7010	47.6419	0.12	43.1120	43.4273	-0.73
20	4	50.3530	50.3811	-0.06	45.4360	45.7241	-0.63
21	4	53.0080	53.1644	-0.30	47.7620	47.7925	-0.06
22	4	55.6660	55.4955	0.31	50.0880	50.0715	0.03
23	4	58.3260	58.3257	0.00	52.4160	52.4773	-0.12
24	4	60.9890	60.8160	0.28	54.7440	54.7639	-0.04
25	4	63.6540	63.5012	0.24	57.0720	57.0633	0.02
26	4	66.3210	66.3417	-0.03	59.4020	59.4271	-0.04
27	4	68.9900	68.9383	0.07	61.7320	61.7727	-0.07
28	4	71.6600	71.5541	0.15	64.0620	64.0466	0.02
29	4	74.3330	74.5249	-0.26	66.3930	66.3358	0.09
30	4	77.0060	76.9724	0.04	68.7250	68.8097	-0.12
2	5	4.0000	3.9698	0.75	4.0000	3.9698	0.75
3	5	6.3750	6.3959	-0.33	6.3750	6.3959	-0.33
4	5	8.8240	8.8492	-0.29	8.8240	8.8492	-0.29

		AMSCRA WITH COL.RESOL.			AMSCRA WITH SEP.RESOL.		
n	b	analy.	sim.	error %	analy.	sim.	error %
5	5	11.3260	11.3625	-0.32	11.3260	11.3315	-0.05
6	5	13.8210	13.8105	0.08	13.0730	13.0699	0.02
7	5	16.3770	16.3929	-0.10	15.4300	15.4430	-0.08
8	5	18.9390	18.9523	-0.07	17.9070	17.9005	0.04
9	5	21.5180	21.4957	0.10	20.3250	20.2573	0.33
10	5	24.1100	24.1263	-0.07	22.6560	22.6339	0.10
11	5	26.7130	26.6784	0.13	24.9480	24.8191	0.52
12	5	29.3250	29.3221	0.01	27.2610	27.2087	0.19
13	5	31.9440	31.9623	-0.06	29.6080	29.7040	-0.32
14	5	34.5700	34.6912	-0.35	31.9690	31.9432	0.08
15	5	37.2020	37.1089	0.25	34.3270	34.3741	-0.14
16	5	39.8390	39.8659	-0.07	36.6760	36.6813	-0.01
17	5	42.4800	42.5752	-0.22	39.0220	39.0413	-0.05
18	5	45.1260	44.9835	0.32	41.3710	41.2751	0.23
19	5	47.7750	47.7523	0.05	43.7240	43.9842	-0.60
20	5	50.4280	50.3917	0.07	46.0800	46.2784	-0.43
21	5	53.0830	53.2127	-0.24	48.4360	48.3495	0.18
22	5	55.7420	55.6015	0.25	50.7920	50.8447	-0.10
23	5	58.4030	58.1979	0.35	53.1480	53.1672	-0.04
24	5	61.0660	61.1881	-0.20	55.5050	55.4857	0.03
25	5	63.7310	63.6266	0.16	57.8640	57.7555	0.19
26	5	66.3990	66.4654	-0.10	60.2220	60.2132	0.01
27	5	69.0680	69.0211	0.07	62.5820	62.3887	0.31
28	5	71.7390	71.6242	0.16	64.9420	64.9576	-0.02
29	5	74.4110	74.4350	-0.03	67.3020	67.2597	0.06
30	5	77.0850	77.0679	0.02	69.6630	69.7066	-0.06
2	6	4.0000	4.0440	-1.10	4.0000	4.0440	-1.10
3	6	6.3750	6.4127	-0.59	6.3750	6.4127	-0.59
4	6	8.8240	8.7997	0.28	8.8240	8.7997	0.28
5	6	11.3260	11.3239	0.02	11.3260	11.3239	0.02
6	6	13.8650	13.9101	-0.33	13.8650	13.8161	0.35
7	6	16.3990	16.3938	0.03	15.6020	15.5811	0.13
8	6	18.9720	19.0009	-0.15	17.9520	17.9169	0.20
9	6	21.5550	21.6529	-0.45	20.4430	20.4490	-0.03
10	6	24.1490	24.1651	-0.07	22.9240	22.9553	-0.14
11	6	26.7530	26.7508	0.01	25.3420	25.2817	0.24
12	6	29.3670	29.3753	-0.03	27.6900	27.7113	-0.08
13	6	31.9880	31.8991	0.28	30.0080	29.9976	0.03
14	6	34.6150	34.5688	0.13	32.3440	32.2936	0.16
15	6	37.2480	37.3251	-0.21	34.7160	34.8387	-0.35
16	6	39.8860	39.8968	-0.03	37.1100	37.0553	0.15
17	6	42.5280	42.4860	0.10	39.5050	39.4722	0.08
18	6	45.1750	45.0991	0.17	41.8900	41.9454	-0.13
19	6	47.8250	47.6649	0.33	44.2650	44.4469	-0.41
20	6	50.4780	50.5287	-0.10	46.6380	46.7204	-0.18
21	6	53.1340	53.0897	0.08	49.0140	48.9711	0.09
22	6	55.7930	55.7698	0.04	51.3960	51.2929	0.20
23	6	58.4540	58.4708	-0.03	53.7810	53.6921	0.17
24	6	61.1180	61.1139	0.01	56.1660	56.1162	0.09
25	6	63.7840	63.7830	0.00	58.5510	58.5761	-0.04
26	6	66.4510	66.4592	-0.01	60.9350	60.8967	0.06
27	6	69.1210	69.0624	0.08	63.3190	63.3797	-0.10
28	6	71.7920	71.9707	-0.25	65.7050	65.6195	0.13
29	6	74.4650	74.4758	-0.01	68.0910	68.0439	0.07
30	6	77.1390	77.1548	-0.02	70.4770	70.3976	0.11
2	7	4.0000	3.9442	1.40	4.0000	3.9442	1.40
3	7	6.3750	6.3963	-0.33	6.3750	6.3963	-0.33
4	7	8.8240	8.8549	-0.35	8.8240	8.8549	-0.35
5	7	11.3260	11.3238	0.02	11.3260	11.3238	0.02
6	7	13.8650	13.8473	0.13	13.8650	13.8473	0.13
7	7	16.4290	16.4825	-0.33	16.4290	16.4191	0.06
8	7	18.9890	18.9423	0.25	18.1550	18.1623	-0.04
9	7	21.5770	21.6063	-0.14	20.4990	20.5209	-0.11
10	7	24.1750	24.1583	0.07	22.9910	23.0030	-0.05

		AMSCRA WITH COL.RESOL.			AMSCRA WITH SEP.RESOL.		
n	b	analy.	sim.	error %	analy.	sim.	error %
11	7	26.7810	26.7807	0.00	25.4980	25.5105	-0.05
12	7	29.3960	29.3625	0.11	27.9810	27.9545	0.09
13	7	32.0180	31.9887	0.09	30.4080	30.3487	0.19
14	7	34.6470	34.7013	-0.16	32.7740	32.7311	0.13
15	7	37.2810	37.2536	0.07	35.1120	35.1639	-0.15
16	7	39.9200	39.9223	-0.01	37.4660	37.3843	0.22
17	7	42.5630	42.6224	-0.14	39.8550	39.9412	-0.22
18	7	45.2100	45.1332	0.17	42.2730	42.2158	0.14
19	7	47.8610	47.8461	0.03	44.7000	44.9206	-0.49
20	7	50.5150	50.5617	-0.09	47.1190	47.3477	-0.49
21	7	53.1720	53.1922	-0.04	49.5240	49.4391	0.17
22	7	55.8310	55.8340	-0.01	51.9200	51.9637	-0.08
23	7	58.4930	58.5160	-0.04	54.3150	54.3738	-0.11
24	7	61.1570	61.0750	0.13	56.7140	56.6577	0.10
25	7	63.8230	63.7242	0.15	59.1190	59.1205	0.00
26	7	66.4910	66.4237	0.10	61.5280	61.3773	0.24
27	7	69.1610	69.1377	0.03	63.9390	63.8974	0.07
28	7	71.8330	71.8255	0.01	66.3490	66.3711	-0.03
29	7	74.5060	74.5145	-0.01	68.7570	68.8084	-0.07
30	7	77.1800	77.2077	-0.04	71.1640	71.0568	0.15
2	8	4.0000	4.0014	-0.04	4.0000	4.0014	-0.04
3	8	6.3750	6.3733	0.03	6.3750	6.3733	0.03
4	8	8.8240	8.8667	-0.48	8.8240	8.8667	-0.48
5	8	11.3260	11.3619	-0.32	11.3260	11.3619	-0.32
6	8	13.8650	13.8907	-0.19	13.8650	13.8907	-0.19
7	8	16.4290	16.4669	-0.23	16.4290	16.4669	-0.23
8	8	19.0120	18.9938	0.10	19.0120	19.0385	-0.14
9	8	21.5910	21.6655	-0.35	20.7310	20.7489	-0.09
10	8	24.1910	24.1826	0.03	23.0690	23.0591	0.04
11	8	26.8010	26.7803	0.08	25.5580	25.5134	0.17
12	8	29.4180	29.3890	0.10	28.0740	28.0721	0.01
13	8	32.0410	32.0009	0.13	30.5900	30.5409	0.16
14	8	34.6710	34.5986	0.21	33.0810	33.0492	0.10
15	8	37.3060	37.3436	-0.10	35.5200	35.3415	0.50
16	8	39.9460	39.8674	0.20	37.9020	37.9247	-0.06
17	8	42.5900	42.5095	0.19	40.2580	40.2996	-0.10
18	8	45.2380	45.2005	0.08	42.6250	42.5900	0.08
19	8	47.8890	47.8100	0.17	45.0260	45.0647	-0.09
20	8	50.5440	50.4773	0.13	47.4610	47.5253	-0.14
21	8	53.2010	53.2589	-0.11	49.9120	49.8520	0.12
22	8	55.8610	55.9313	-0.13	52.3630	52.1677	0.37
23	8	58.5240	58.5583	-0.06	54.8000	54.8027	0.00
24	8	61.1880	61.0296	0.26	57.2240	57.4703	-0.43
25	8	63.8550	63.8077	0.07	59.6380	59.5725	0.11
26	8	66.5230	66.4443	0.12	62.0500	62.0501	0.00
27	8	69.1930	69.2580	-0.09	64.4680	64.5171	-0.08
28	8	71.8650	71.8147	0.07	66.8920	66.8407	0.08
29	8	74.5390	74.5865	-0.06	69.3220	69.4233	-0.15
30	8	77.2130	77.0777	0.18	71.7550	71.7903	-0.05
2	9	4.0000	4.0216	-0.54	4.0000	4.0216	-0.54
3	9	6.3750	6.3553	0.31	6.3750	6.3553	0.31
4	9	8.8240	8.7917	0.37	8.8240	8.7917	0.37
5	9	11.3260	11.3707	-0.39	11.3260	11.3707	-0.39
6	9	13.8650	13.7839	0.58	13.8650	13.7839	0.58
7	9	16.4290	16.4121	0.10	16.4290	16.4121	0.10
8	9	19.0120	19.1109	-0.52	19.0120	19.1109	-0.52
9	9	21.6100	21.5463	0.29	21.6100	21.7097	-0.46
10	9	24.2040	24.1631	0.17	23.3230	23.2420	0.35
11	9	26.8140	26.8383	-0.09	25.6560	25.5744	0.32
12	9	29.4340	29.4431	-0.03	28.1420	28.1855	-0.15
13	9	32.0590	31.9978	0.19	30.6590	30.5489	0.36
14	9	34.6900	34.7021	-0.03	33.1890	33.1548	0.10
15	9	37.3260	37.2088	0.31	35.7170	35.6381	0.22
16	9	39.9670	39.9305	0.09	38.2190	38.1212	0.26

		AMSCRA WITH COL.RESOL.			AMSCRA WITH SEP.RESOL.		
n	b	analy.	sim.	error %	analy.	sim.	error %
17	9	42.6120	42.6061	0.01	40.6690	40.5955	0.18
18	9	45.2600	45.2310	0.06	43.0650	43.1311	-0.15
19	9	47.9120	47.9860	-0.15	45.4350	45.4011	0.07
20	9	50.5670	50.5538	0.03	47.8140	48.0123	-0.41
21	9	53.2250	53.2441	-0.04	50.2230	50.3047	-0.16
22	9	55.8860	55.7775	0.19	52.6690	52.7274	-0.11
23	9	58.5490	58.6099	-0.10	55.1380	55.1126	0.05
24	9	61.2130	61.0843	0.21	57.6130	57.6300	-0.03
25	9	63.8800	63.8699	0.02	60.0810	60.1379	-0.09
26	9	66.5490	66.6627	-0.17	62.5340	62.5023	0.05
27	9	69.2200	69.2328	-0.02	64.9730	64.9539	0.03
28	9	71.8920	71.8579	0.05	67.4020	67.4025	0.00
29	9	74.5660	74.5361	0.04	69.8300	69.8393	-0.01
30	9	77.2410	77.0925	0.19	72.2620	72.1780	0.12
2	10	4.0000	4.0260	-0.65	4.0000	4.0260	-0.65
3	10	6.3750	6.3711	0.06	6.3750	6.3711	0.06
4	10	8.8240	8.8385	-0.16	8.8240	8.8385	-0.16
5	10	11.3260	11.3325	-0.06	11.3260	11.3325	-0.06
6	10	13.8650	13.8257	0.28	13.8650	13.8257	0.28
7	10	16.4290	16.3893	0.24	16.4290	16.3893	0.24
8	10	19.0120	18.9936	0.10	19.0120	18.9936	0.10
9	10	21.6100	21.5441	0.31	21.6100	21.5441	0.31
10	10	24.2200	24.2977	-0.32	24.2200	24.1638	0.23
11	10	26.8250	26.8485	-0.09	25.9280	25.8498	0.30
12	10	29.4440	29.4524	-0.03	28.2570	28.2104	0.16
13	10	32.0720	32.1475	-0.24	30.7400	30.7643	-0.08
14	10	34.7050	34.7284	-0.07	33.2570	33.2043	0.16
15	10	37.3420	37.3076	0.09	35.7910	35.7181	0.20
16	10	39.9830	39.8744	0.27	38.3360	38.3723	-0.09
17	10	42.6290	42.6412	-0.03	40.8780	40.9051	-0.07
18	10	45.2780	45.3559	-0.17	43.3880	43.4279	-0.09
19	10	47.9310	47.8315	0.21	45.8480	45.8077	0.09
20	10	50.5870	50.6351	-0.10	48.2570	48.3721	-0.24
21	10	53.2450	53.2182	0.05	50.6390	50.7431	-0.21
22	10	55.9060	56.0428	-0.24	53.0270	52.9019	0.24
23	10	58.5690	58.5759	-0.01	55.4430	55.4062	0.07
24	10	61.2350	61.3643	-0.21	57.8950	57.9057	-0.02
25	10	63.9020	63.9335	-0.05	60.3760	60.3765	0.00
26	10	66.5710	66.5663	0.01	62.8710	62.7926	0.12
27	10	69.2420	69.1466	0.14	65.3640	65.3269	0.06
28	10	71.9150	71.9046	0.01	67.8470	67.9287	-0.12
29	10	74.5890	74.6443	-0.07	70.3130	70.3165	0.00
30	10	77.2640	77.3224	-0.08	72.7640	72.7935	-0.04
2	11	4.0000	3.9728	0.68	4.0000	3.9728	0.68
3	11	6.3750	6.3690	0.09	6.3750	6.3690	0.09
4	11	8.8240	8.8071	0.19	8.8240	8.8071	0.19
5	11	11.3260	11.3319	-0.05	11.3260	11.3319	-0.05
6	11	13.8650	13.8525	0.09	13.8650	13.8525	0.09
7	11	16.4290	16.4269	0.01	16.4290	16.4269	0.01
8	11	19.0120	19.0631	-0.27	19.0120	19.0631	-0.27
9	11	21.6100	21.6614	-0.24	21.6100	21.6614	-0.24
10	11	24.2200	24.1098	0.45	24.2200	24.1098	0.45
11	11	26.8390	26.9795	-0.52	26.8390	26.7786	0.23
12	11	29.4550	29.4293	0.09	28.5440	28.6268	-0.29
13	11	32.0810	32.0671	0.04	30.8700	30.8319	0.12
14	11	34.7150	34.7472	-0.09	33.3500	33.3456	0.01
15	11	37.3540	37.2023	0.41	35.8660	35.8197	0.13
16	11	39.9970	39.8043	0.48	38.4010	38.4230	-0.06
17	11	42.6430	42.5873	0.13	40.9530	40.9235	0.07
18	11	45.2930	45.2657	0.06	43.5140	43.4819	0.07
19	11	47.9470	47.9637	-0.03	46.0660	46.0915	-0.06
20	11	50.6030	50.5661	0.07	48.5840	48.6764	-0.19
21	11	53.2620	53.3267	-0.12	51.0520	51.0160	0.07
22	11	55.9230	55.9687	-0.08	53.4710	53.4683	0.00

		AMSCRA WITH COL.RESOL.			AMSCRA WITH SEP.RESOL.		
n	b	analy.	sim.	error %	analy.	sim.	error %
23	11	58.5870	58.6368	-0.08	55.8640	56.0148	-0.27
24	11	61.2530	61.2444	0.01	58.2600	58.2944	-0.06
25	11	63.9210	63.9048	0.03	60.6820	60.7349	-0.09
26	11	66.5900	66.5579	0.05	63.1380	63.1547	-0.03
27	11	69.2610	69.3272	-0.10	65.6260	65.6104	0.02
28	11	71.9340	72.0401	-0.15	68.1340	68.2718	-0.20
29	11	74.6080	74.6351	-0.04	70.6480	70.5189	0.18
30	11	77.2840	77.3722	-0.11	73.1560	73.2028	-0.06
2	12	4.0000	3.9694	0.76	4.0000	3.9694	0.76
3	12	6.3750	6.3699	0.08	6.3750	6.3699	0.08
4	12	8.8240	8.8443	-0.23	8.8240	8.8443	-0.23
5	12	11.3260	11.2723	0.47	11.3260	11.2723	0.47
6	12	13.8650	13.8144	0.36	13.8650	13.8144	0.36
7	12	16.4290	16.3837	0.28	16.4290	16.3837	0.28
8	12	19.0120	19.0180	-0.03	19.0120	19.0180	-0.03
9	12	21.6100	21.6073	0.01	21.6100	21.6073	0.01
10	12	24.2200	24.1427	0.32	24.2200	24.1427	0.32
11	12	26.8390	26.9347	-0.36	26.8390	26.9347	-0.36
12	12	29.4670	29.4177	0.17	29.4670	29.6259	-0.54
13	12	32.0900	32.1104	-0.06	31.1690	31.0743	0.30
14	12	34.7230	34.6957	0.08	33.4910	33.5905	-0.30
15	12	37.3630	37.3601	0.01	35.9690	35.9331	0.10
16	12	40.0080	39.9903	0.04	38.4850	38.6087	-0.32
17	12	42.6550	42.5627	0.22	41.0200	41.0209	0.00
18	12	45.3060	45.2287	0.17	43.5730	43.5859	-0.03
19	12	47.9600	47.9495	0.02	46.1430	46.1340	0.02
20	12	50.6160	50.6197	-0.01	48.7180	48.6408	0.16
21	12	53.2760	53.3142	-0.07	51.2780	51.1611	0.23
22	12	55.9380	55.9291	0.02	53.8020	53.7607	0.08
23	12	58.6020	58.6113	-0.02	56.2770	56.1575	0.21
24	12	61.2680	61.2597	0.01	58.7050	58.6593	0.08
25	12	63.9360	63.9912	-0.09	61.1070	61.1585	-0.08
26	12	66.6060	66.6452	-0.06	63.5110	63.5188	-0.01
27	12	69.2780	69.2917	-0.02	65.9370	65.8965	0.06
28	12	71.9510	71.8452	0.15	68.3960	68.3252	0.10
29	12	74.6250	74.5171	0.14	70.8890	70.7987	0.13
30	12	77.3010	77.2760	0.03	73.4050	73.3567	0.07
2	13	4.0000	4.0414	-1.03	4.0000	4.0414	-1.03
3	13	6.3750	6.3511	0.37	6.3750	6.3511	0.37
4	13	8.8240	8.8411	-0.19	8.8240	8.8411	-0.19
5	13	11.3260	11.3119	0.12	11.3260	11.3119	0.12
6	13	13.8650	13.7887	0.55	13.8650	13.7887	0.55
7	13	16.4290	16.4225	0.04	16.4290	16.4225	0.04
8	13	19.0120	19.0665	-0.29	19.0120	19.0665	-0.29
9	13	21.6100	21.6359	-0.12	21.6100	21.6359	-0.12
10	13	24.2200	24.2178	0.01	24.2200	24.2178	0.01
11	13	26.8390	26.8648	-0.10	26.8390	26.8648	-0.10
12	13	29.4670	29.3635	0.35	29.4670	29.3635	0.35
13	13	32.1020	32.0365	0.20	32.1020	32.0719	0.09
14	13	34.7320	34.6529	0.23	33.8010	33.8064	-0.02
15	13	37.3710	37.3246	0.12	36.1200	36.1194	0.00
16	13	40.0150	40.0479	-0.08	38.5970	38.6293	-0.08
17	13	42.6650	42.6833	-0.04	41.1120	40.9966	0.28
18	13	45.3170	45.2873	0.07	43.6460	43.7359	-0.21
19	13	47.9710	48.0495	-0.16	46.2000	46.2234	-0.05
20	13	50.6280	50.5307	0.19	48.7730	48.7790	-0.01
21	13	53.2880	53.3095	-0.04	51.3580	51.3453	0.02
22	13	55.9500	55.9251	0.04	53.9430	53.8600	0.15
23	13	58.6150	58.6247	-0.02	56.5090	56.3869	0.22
24	13	61.2820	61.1675	0.19	59.0380	59.0377	0.00
25	13	63.9500	63.9469	0.00	61.5190	61.5198	0.00
26	13	66.6200	66.6809	-0.09	63.9550	63.8915	0.10
27	13	69.2920	69.3309	-0.06	66.3650	66.3531	0.02
28	13	71.9650	71.9113	0.07	68.7760	68.6901	0.12

		AMSCRA WITH COL.RESOL.			AMSCRA WITH SEP.RESOL.		
n	b	analy.	sim.	error %	analy.	sim.	error %
29	13	74.6400	74.7379	-0.13	71.2060	71.1089	0.14
30	13	77.3160	77.2315	0.11	73.6680	73.6153	0.07
2	14	4.0000	3.9842	0.39	4.0000	3.9842	0.39
3	14	6.3750	6.3875	-0.20	6.3750	6.3875	-0.20
4	14	8.8240	8.8495	-0.29	8.8240	8.8495	-0.29
5	14	11.3260	11.3129	0.12	11.3260	11.3129	0.12
6	14	13.8650	13.8345	0.22	13.8650	13.8345	0.22
7	14	16.4290	16.4681	-0.24	16.4290	16.4681	-0.24
8	14	19.0120	18.9883	0.12	19.0120	18.9883	0.12
9	14	21.6100	21.6122	-0.01	21.6100	21.6122	-0.01
10	14	24.2200	24.2828	-0.26	24.2200	24.2828	-0.26
11	14	26.8390	26.8184	0.08	26.8390	26.8184	0.08
12	14	29.4670	29.4825	-0.05	29.4670	29.4825	-0.05
13	14	32.1020	32.2155	-0.35	32.1020	32.2155	-0.35
14	14	34.7430	34.8406	-0.28	34.7430	34.7859	-0.12
15	14	37.3790	37.2675	0.30	36.4390	36.3727	0.18
16	14	40.0220	39.9481	0.18	38.7560	38.7257	0.08
17	14	42.6710	42.6800	-0.02	41.2310	41.2184	0.03
18	14	45.3240	45.3353	-0.02	43.7450	43.7914	-0.11
19	14	47.9800	47.9741	0.01	46.2790	46.3254	-0.10
20	14	50.6380	50.5864	0.10	48.8340	48.8793	-0.09
21	14	53.2990	53.3801	-0.15	51.4070	51.4613	-0.11
22	14	55.9610	55.9608	0.00	53.9960	53.9458	0.09
23	14	58.6260	58.5787	0.08	56.5930	56.6448	-0.09
24	14	61.2930	61.2609	0.05	59.1860	59.1989	-0.02
25	14	63.9620	63.9058	0.09	61.7570	61.8444	-0.14
26	14	66.6330	66.4695	0.25	64.2900	64.3417	-0.08
27	14	69.3050	69.2235	0.12	66.7750	66.6814	0.14
28	14	71.9780	71.7681	0.29	69.2180	69.2511	-0.05
29	14	74.6530	74.7163	-0.08	71.6370	71.6469	-0.01
30	14	77.3300	77.3390	-0.01	74.0530	74.0435	0.01
2	15	4.0000	3.9942	0.14	4.0000	3.9942	0.14
3	15	6.3750	6.4019	-0.42	6.3750	6.4019	-0.42
4	15	8.8240	8.7951	0.33	8.8240	8.7951	0.33
5	15	11.3260	11.3209	0.05	11.3260	11.3209	0.05
6	15	13.8650	13.8749	-0.07	13.8650	13.8749	-0.07
7	15	16.4290	16.3761	0.32	16.4290	16.3761	0.32
8	15	19.0120	18.9560	0.29	19.0120	18.9560	0.29
9	15	21.6100	21.6313	-0.10	21.6100	21.6313	-0.10
10	15	24.2200	24.0729	0.61	24.2200	24.0729	0.61
11	15	26.8390	26.9298	-0.34	26.8390	26.9298	-0.34
12	15	29.4670	29.4383	0.10	29.4670	29.4383	0.10
13	15	32.1020	32.0998	0.01	32.1020	32.0998	0.01
14	15	34.7430	34.7794	-0.10	34.7430	34.7794	-0.10
15	15	37.3890	37.3360	0.14	37.3890	37.3576	0.08
16	15	40.0300	40.0667	-0.09	39.0830	39.0729	0.03
17	15	42.6780	42.7198	-0.10	41.3980	41.5140	-0.28
18	15	45.3300	45.3140	0.04	43.8710	43.9234	-0.12
19	15	47.9870	48.0100	-0.05	46.3840	46.4702	-0.19
20	15	50.6460	50.5455	0.20	48.9180	48.9380	-0.04
21	15	53.3080	53.2872	0.04	51.4730	51.5549	-0.16
22	15	55.9710	55.9445	0.05	54.0460	54.0949	-0.09
23	15	58.6360	58.5617	0.13	56.6370	56.6313	0.01
24	15	61.3030	61.3889	-0.14	59.2390	59.1183	0.20
25	15	63.9720	63.8924	0.12	61.8450	61.9213	-0.12
26	15	66.6430	66.7209	-0.12	64.4440	64.4689	-0.04
27	15	69.3160	69.2187	0.14	67.0180	66.9415	0.11
28	15	71.9900	71.8865	0.14	69.5540	69.4720	0.12
29	15	74.6650	74.6969	-0.04	72.0440	71.9529	0.13
30	15	77.3420	77.4788	-0.18	74.4940	74.5091	-0.02
2	16	4.0000	3.9748	0.63	4.0000	3.9748	0.63
3	16	6.3750	6.3709	0.06	6.3750	6.3709	0.06
4	16	8.8240	8.7859	0.43	8.8240	8.7859	0.43
5	16	11.3260	11.2937	0.29	11.3260	11.2937	0.29

		AMSCRA WITH COL.RESOL.			AMSCRA WITH SEP.RESOL.		
n	b	analy.	sim.	error %	analy.	sim.	error %
6	16	13.8650	13.7839	0.58	13.8650	13.7839	0.58
7	16	16.4290	16.4537	-0.15	16.4290	16.4537	-0.15
8	16	19.0120	19.0258	-0.07	19.0120	19.0258	-0.07
9	16	21.6100	21.5665	0.20	21.6100	21.5665	0.20
10	16	24.2200	24.2249	-0.02	24.2200	24.2249	-0.02
11	16	26.8390	26.7674	0.27	26.8390	26.7674	0.27
12	16	29.4670	29.4043	0.21	29.4670	29.4043	0.21
13	16	32.1020	32.0181	0.26	32.1020	32.0181	0.26
14	16	34.7430	34.8316	-0.25	34.7430	34.8316	-0.25
15	16	37.3890	37.3389	0.13	37.3890	37.3389	0.13
16	16	40.0390	40.0077	0.08	40.0390	39.9877	0.13
17	16	42.6850	42.7270	-0.10	41.7320	41.7702	-0.09
18	16	45.3360	45.3285	0.02	44.0450	44.0300	0.03
19	16	47.9930	47.9923	0.00	46.5160	46.5815	-0.14
20	16	50.6520	50.6277	0.05	49.0280	49.1786	-0.31
21	16	53.3150	53.4001	-0.16	51.5620	51.6522	-0.17
22	16	55.9790	56.0015	-0.04	54.1170	54.1943	-0.14
23	16	58.6450	58.7219	-0.13	56.6900	56.6685	0.04
24	16	61.3120	61.2959	0.03	59.2810	59.2042	0.13
25	16	63.9810	63.7885	0.30	61.8850	61.8553	0.05
26	16	66.6520	66.5467	0.16	64.4980	64.5609	-0.10
27	16	69.3250	69.3318	-0.01	67.1110	67.0815	0.04
28	16	72.0000	71.9720	0.04	69.7140	69.5978	0.17
29	16	74.6760	74.7566	-0.11	72.2910	72.2861	0.01
30	16	77.3520	77.2010	0.20	74.8300	75.0517	-0.30
2	17	4.0000	3.9788	0.53	4.0000	3.9788	0.53
3	17	6.3750	6.4153	-0.63	6.3750	6.4153	-0.63
4	17	8.8240	8.8475	-0.27	8.8240	8.8475	-0.27
5	17	11.3260	11.3327	-0.06	11.3260	11.3327	-0.06
6	17	13.8650	13.8959	-0.22	13.8650	13.8959	-0.22
7	17	16.4290	16.3837	0.28	16.4290	16.3837	0.28
8	17	19.0120	19.0779	-0.35	19.0120	19.0779	-0.35
9	17	21.6100	21.5979	0.06	21.6100	21.5979	0.06
10	17	24.2200	24.2388	-0.08	24.2200	24.2388	-0.08
11	17	26.8390	26.7867	0.19	26.8390	26.7867	0.19
12	17	29.4670	29.5175	-0.17	29.4670	29.5175	-0.17
13	17	32.1020	32.1311	-0.09	32.1020	32.1311	-0.09
14	17	34.7430	34.7372	0.02	34.7430	34.7372	0.02
15	17	37.3890	37.4634	-0.20	37.3890	37.4634	-0.20
16	17	40.0390	40.0439	-0.01	40.0390	40.0439	-0.01
17	17	42.6930	42.6357	0.13	42.6930	42.6294	0.15
18	17	45.3430	45.3745	-0.07	44.3850	44.4817	-0.22
19	17	47.9980	48.0647	-0.14	46.6960	46.6714	0.05
20	17	50.6570	50.5949	0.12	49.1650	49.1567	0.02
21	17	53.3200	53.3996	-0.15	51.6770	51.7073	-0.06
22	17	55.9850	56.0259	-0.07	54.2110	54.1888	0.04
23	17	58.6520	58.6562	-0.01	56.7650	56.7577	0.01
24	17	61.3200	61.3985	-0.13	59.3380	59.2929	0.08
25	17	63.9900	63.9960	-0.01	61.9290	62.1230	-0.31
26	17	66.6610	66.6590	0.00	64.5340	64.5305	0.01
27	17	69.3340	69.2069	0.18	67.1490	67.0312	0.18
28	17	72.0080	71.9039	0.14	69.7700	69.9653	-0.28
29	17	74.6850	74.6701	0.02	72.3900	72.4364	-0.06
30	17	77.3620	77.4678	-0.14	74.9960	74.9060	0.12
2	18	4.0000	3.9592	1.02	4.0000	3.9592	1.02
3	18	6.3750	6.3514	0.37	6.3750	6.3514	0.37
4	18	8.8240	8.7985	0.29	8.8240	8.7985	0.29
5	18	11.3260	11.4203	-0.83	11.3260	11.4203	-0.83
6	18	13.8650	13.8753	-0.07	13.8650	13.8753	-0.07
7	18	16.4290	16.4809	-0.32	16.4290	16.4809	-0.32
8	18	19.0120	19.0082	0.02	19.0120	19.0082	0.02
9	18	21.6100	21.6239	-0.06	21.6100	21.6239	-0.06
10	18	24.2200	24.2411	-0.09	24.2200	24.2411	-0.09
11	18	26.8390	26.8377	0.00	26.8390	26.8377	0.00

		AMSCRA WITH COL.RESOL.			AMSCRA WITH SEP.RESOL.		
n	b	analy.	sim.	error %	analy.	sim.	error %
12	18	29.4670	29.4303	0.12	29.4670	29.4303	0.12
13	18	32.1020	32.2028	-0.31	32.1020	32.2028	-0.31
14	18	34.7430	34.7054	0.11	34.7430	34.7054	0.11
15	18	37.3890	37.4514	-0.17	37.3890	37.4514	-0.17
16	18	40.0390	39.9645	0.19	40.0390	39.9645	0.19
17	18	42.6930	42.7507	-0.14	42.6930	42.7507	-0.14
18	18	45.3510	45.2560	0.21	45.3510	45.2642	0.19
19	18	48.0050	48.0243	-0.04	47.0410	47.1677	-0.27
20	18	50.6630	50.7045	-0.08	49.3510	49.3899	-0.08
21	18	53.3250	53.3509	-0.05	51.8190	51.7802	0.07
22	18	55.9900	56.1382	-0.26	54.3300	54.3025	0.05
23	18	58.6570	58.6753	-0.03	56.8630	56.8732	-0.02
24	18	61.3270	61.1960	0.21	59.4170	59.4931	-0.13
25	18	63.9970	63.9642	0.05	61.9910	62.0077	-0.03
26	18	66.6690	66.5892	0.12	64.5810	64.4677	0.18
27	18	69.3420	69.3522	-0.01	67.1860	67.1588	0.04
28	18	72.0160	72.0932	-0.11	69.8020	69.7365	0.09
29	18	74.6920	74.7022	-0.01	72.4270	72.3984	0.04
30	18	77.3700	77.2761	0.12	75.0550	75.0688	-0.02
2	19	4.0000	4.0150	-0.38	4.0000	4.0150	-0.38
3	19	6.3750	6.3718	0.05	6.3750	6.3718	0.05
4	19	8.8240	8.8636	-0.45	8.8240	8.8636	-0.45
5	19	11.3260	11.3247	0.01	11.3260	11.3247	0.01
6	19	13.8650	13.8360	0.21	13.8650	13.8360	0.21
7	19	16.4290	16.4415	-0.08	16.4290	16.4415	-0.08
8	19	19.0120	18.9878	0.13	19.0120	18.9878	0.13
9	19	21.6100	21.6213	-0.05	21.6100	21.6213	-0.05
10	19	24.2200	24.1885	0.13	24.2200	24.1885	0.13
11	19	26.8390	26.8106	0.11	26.8390	26.8106	0.11
12	19	29.4670	29.4433	0.08	29.4670	29.4433	0.08
13	19	32.1020	32.1021	0.00	32.1020	32.1021	0.00
14	19	34.7430	34.7859	-0.12	34.7430	34.7859	-0.12
15	19	37.3890	37.4260	-0.10	37.3890	37.4260	-0.10
16	19	40.0390	40.0826	-0.11	40.0390	40.0826	-0.11
17	19	42.6930	42.8023	-0.26	42.6930	42.8023	-0.26
18	19	45.3510	45.3747	-0.05	45.3510	45.3747	-0.05
19	19	48.0120	48.0679	-0.12	48.0120	47.9293	0.17
20	19	50.6690	50.6873	-0.04	49.7010	49.7630	-0.12
21	19	53.3300	53.4329	-0.19	52.0090	52.0435	-0.07
22	19	55.9940	56.0187	-0.04	54.4760	54.4767	0.00
23	19	58.6620	58.6231	0.07	56.9860	57.0877	-0.18
24	19	61.3320	61.3765	-0.07	59.5190	59.3901	0.22
25	19	64.0030	63.9342	0.11	62.0730	62.1361	-0.10
26	19	66.6750	66.7415	-0.10	64.6460	64.6566	-0.02
27	19	69.3490	69.2997	0.07	67.2370	67.2411	-0.01
28	19	72.0240	72.0637	-0.06	69.8420	69.8663	-0.03
29	19	74.7000	74.6957	0.01	72.4580	72.5275	-0.10
30	19	77.3770	77.2103	0.22	75.0840	74.9739	0.15
2	20	4.0000	3.9940	0.15	4.0000	3.9940	0.15
3	20	6.3750	6.3979	-0.36	6.3750	6.3979	-0.36
4	20	8.8240	8.8068	0.19	8.8240	8.8068	0.19
5	20	11.3260	11.3479	-0.19	11.3260	11.3479	-0.19
6	20	13.8650	13.9227	-0.42	13.8650	13.9227	-0.42
7	20	16.4290	16.4237	0.03	16.4290	16.4237	0.03
8	20	19.0120	18.9830	0.15	19.0120	18.9830	0.15
9	20	21.6100	21.6097	0.00	21.6100	21.6097	0.00
10	20	24.2200	24.2136	0.03	24.2200	24.2136	0.03
11	20	26.8390	26.7537	0.32	26.8390	26.7537	0.32
12	20	29.4670	29.4692	-0.01	29.4670	29.4692	-0.01
13	20	32.1020	32.1579	-0.17	32.1020	32.1579	-0.17
14	20	34.7430	34.7328	0.03	34.7430	34.7328	0.03
15	20	37.3890	37.3635	0.07	37.3890	37.3635	0.07
16	20	40.0390	40.0405	0.00	40.0390	40.0405	0.00
17	20	42.6930	42.6678	0.06	42.6930	42.6678	0.06

		AMSCRA WITH COL.RESOL.			AMSCRA WITH SEP.RESOL.		
n	b	analy.	sim.	error %	analy.	sim.	error %
18	20	45.3510	45.3329	0.04	45.3510	45.3329	0.04
19	20	48.0120	48.0762	-0.13	48.0120	48.0762	-0.13
20	20	50.6760	50.6743	0.00	50.6760	50.6669	0.02
21	20	53.3360	53.4503	-0.21	52.3640	52.3366	0.05
22	20	55.9990	55.9161	0.15	54.6710	54.6869	-0.03
23	20	58.6660	58.7815	-0.20	57.1360	57.1769	-0.07
24	20	61.3360	61.3839	-0.08	59.6460	59.5434	0.17
25	20	64.0070	64.0316	-0.04	62.1780	62.0370	0.23
26	20	66.6810	66.6596	0.03	64.7320	64.7457	-0.02
27	20	69.3550	69.3886	-0.05	67.3050	67.1619	0.21
28	20	72.0310	72.1233	-0.13	69.8950	69.8831	0.02
29	20	74.7070	74.7163	-0.01	72.5000	72.3895	0.15
30	20	77.3840	77.4971	-0.15	75.1170	75.1253	-0.01
2	21	4.0000	3.9882	0.30	4.0000	3.9882	0.30
3	21	6.3750	6.3961	-0.33	6.3750	6.3961	-0.33
4	21	8.8240	8.8190	0.06	8.8240	8.8190	0.06
5	21	11.3260	11.2824	0.38	11.3260	11.2824	0.38
6	21	13.8650	13.9041	-0.28	13.8650	13.9041	-0.28
7	21	16.4290	16.3589	0.43	16.4290	16.3589	0.43
8	21	19.0120	19.0549	-0.23	19.0120	19.0549	-0.23
9	21	21.6100	21.6449	-0.16	21.6100	21.6449	-0.16
10	21	24.2200	24.1641	0.23	24.2200	24.1641	0.23
11	21	26.8390	26.8810	-0.16	26.8390	26.8810	-0.16
12	21	29.4670	29.4867	-0.07	29.4670	29.4867	-0.07
13	21	32.1020	32.0477	0.17	32.1020	32.0477	0.17
14	21	34.7430	34.6727	0.20	34.7430	34.6727	0.20
15	21	37.3890	37.3054	0.22	37.3890	37.3054	0.22
16	21	40.0390	40.0369	0.01	40.0390	40.0369	0.01
17	21	42.6930	42.6970	-0.01	42.6930	42.6970	-0.01
18	21	45.3510	45.2397	0.25	45.3510	45.2397	0.25
19	21	48.0120	47.9555	0.12	48.0120	47.9555	0.12
20	21	50.6760	50.6731	0.01	50.6760	50.6731	0.01
21	21	53.3430	53.2771	0.12	53.3430	53.4056	-0.12
22	21	56.0050	55.9561	0.09	55.0300	54.9945	0.06
23	21	58.6710	58.5072	0.28	57.3350	57.3019	0.06
24	21	61.3400	61.3433	-0.01	59.8000	59.8009	0.00
25	21	64.0110	64.0534	-0.07	62.3080	62.2581	0.08
26	21	66.6850	66.6596	0.04	64.8400	64.8225	0.03
27	21	69.3600	69.4093	-0.07	67.3940	67.3337	0.09
28	21	72.0360	72.1029	-0.09	69.9670	69.9375	0.04
29	21	74.7130	74.6203	0.12	72.5570	72.4546	0.14
30	21	77.3910	77.4328	-0.05	75.1620	75.1059	0.07
2	22	4.0000	4.0122	-0.31	4.0000	4.0122	-0.31
3	22	6.3750	6.3395	0.56	6.3750	6.3395	0.56
4	22	8.8240	8.8463	-0.25	8.8240	8.8463	-0.25
5	22	11.3260	11.2927	0.29	11.3260	11.2927	0.29
6	22	13.8650	13.8301	0.25	13.8650	13.8301	0.25
7	22	16.4290	16.3815	0.29	16.4290	16.3815	0.29
8	22	19.0120	18.9886	0.12	19.0120	18.9886	0.12
9	22	21.6100	21.4551	0.72	21.6100	21.4551	0.72
10	22	24.2200	24.2087	0.05	24.2200	24.2087	0.05
11	22	26.8390	26.7778	0.23	26.8390	26.7778	0.23
12	22	29.4670	29.4333	0.11	29.4670	29.4333	0.11
13	22	32.1020	32.1631	-0.19	32.1020	32.1631	-0.19
14	22	34.7430	34.7809	-0.11	34.7430	34.7809	-0.11
15	22	37.3890	37.3548	0.09	37.3890	37.3548	0.09
16	22	40.0390	40.1309	-0.23	40.0390	40.1309	-0.23
17	22	42.6930	42.6781	0.03	42.6930	42.6781	0.03
18	22	45.3510	45.4117	-0.13	45.3510	45.4117	-0.13
19	22	48.0120	47.9661	0.10	48.0120	47.9661	0.10
20	22	50.6760	50.6198	0.11	50.6760	50.6198	0.11
21	22	53.3430	53.3715	-0.05	53.3430	53.3715	-0.05
22	22	56.0110	55.9586	0.09	56.0110	55.9283	0.15
23	22	58.6760	58.6634	0.02	57.6980	57.7030	-0.01

		AMSCRA WITH COL.RESOL.			AMSCRA WITH SEP.RESOL.		
n	b	analy.	sim.	error %	analy.	sim.	error %
24	22	61.3440	61.3781	-0.06	60.0020	60.0811	-0.13
25	22	64.0150	64.1121	-0.15	62.4650	62.5046	-0.06
26	22	66.6880	66.8118	-0.19	64.9740	64.8161	0.24
27	22	69.3630	69.2135	0.22	67.5050	67.4672	0.06
28	22	72.0400	71.9789	0.08	70.0580	69.9976	0.09
29	22	74.7180	74.7155	0.00	72.6310	72.7547	-0.17
30	22	77.3970	77.4182	-0.03	75.2210	75.2138	0.01
2	23	4.0000	4.0330	-0.82	4.0000	4.0330	-0.82
3	23	6.3750	6.3673	0.12	6.3750	6.3673	0.12
4	23	8.8240	8.8588	-0.39	8.8240	8.8588	-0.39
5	23	11.3260	11.2833	0.38	11.3260	11.2833	0.38
6	23	13.8650	13.8601	0.04	13.8650	13.8601	0.04
7	23	16.4290	16.4031	0.16	16.4290	16.4031	0.16
8	23	19.0120	19.0997	-0.46	19.0120	19.0997	-0.46
9	23	21.6100	21.6934	-0.39	21.6100	21.6934	-0.39
10	23	24.2200	24.1838	0.15	24.2200	24.1838	0.15
11	23	26.8390	26.9070	-0.25	26.8390	26.9070	-0.25
12	23	29.4670	29.4427	0.08	29.4670	29.4427	0.08
13	23	32.1020	32.0869	0.05	32.1020	32.0869	0.05
14	23	34.7430	34.8143	-0.21	34.7430	34.8143	-0.21
15	23	37.3890	37.3881	0.00	37.3890	37.3881	0.00
16	23	40.0390	40.1821	-0.36	40.0390	40.1821	-0.36
17	23	42.6930	42.6275	0.15	42.6930	42.6275	0.15
18	23	45.3510	45.3513	0.00	45.3510	45.3513	0.00
19	23	48.0120	47.9999	0.03	48.0120	47.9999	0.03
20	23	50.6760	50.6228	0.11	50.6760	50.6228	0.11
21	23	53.3430	53.2662	0.14	53.3430	53.2662	0.14
22	23	56.0110	56.0173	-0.01	56.0110	56.0173	-0.01
23	23	58.6820	58.7231	-0.07	58.6820	58.5941	0.15
24	23	61.3490	61.3647	-0.03	60.3680	60.4173	-0.08
25	23	64.0190	64.0451	-0.04	62.6710	62.5765	0.15
26	23	66.6920	66.7471	-0.08	65.1340	65.0678	0.10
27	23	69.3670	69.3670	0.00	67.6410	67.6243	0.02
28	23	72.0430	72.0031	0.06	70.1720	70.0572	0.16
29	23	74.7220	74.6862	0.05	72.7250	72.6034	0.17
30	23	77.4010	77.4572	-0.07	75.2980	75.3237	-0.03
2	24	4.0000	4.0258	-0.64	4.0000	4.0258	-0.64
3	24	6.3750	6.3628	0.19	6.3750	6.3628	0.19
4	24	8.8240	8.8559	-0.36	8.8240	8.8559	-0.36
5	24	11.3260	11.3074	0.16	11.3260	11.3074	0.16
6	24	13.8650	13.8742	-0.07	13.8650	13.8742	-0.07
7	24	16.4290	16.4762	-0.29	16.4290	16.4762	-0.29
8	24	19.0120	19.0015	0.06	19.0120	19.0015	0.06
9	24	21.6100	21.5699	0.19	21.6100	21.5699	0.19
10	24	24.2200	24.1954	0.10	24.2200	24.1954	0.10
11	24	26.8390	26.8357	0.01	26.8390	26.8357	0.01
12	24	29.4670	29.4269	0.14	29.4670	29.4269	0.14
13	24	32.1020	32.0343	0.21	32.1020	32.0343	0.21
14	24	34.7430	34.7241	0.05	34.7430	34.7241	0.05
15	24	37.3890	37.3801	0.02	37.3890	37.3801	0.02
16	24	40.0390	39.9477	0.23	40.0390	39.9477	0.23
17	24	42.6930	42.6843	0.02	42.6930	42.6843	0.02
18	24	45.3510	45.3594	-0.02	45.3510	45.3594	-0.02
19	24	48.0120	47.9876	0.05	48.0120	47.9876	0.05
20	24	50.6760	50.6376	0.08	50.6760	50.6376	0.08
21	24	53.3430	53.3924	-0.09	53.3430	53.3924	-0.09
22	24	56.0110	55.9263	0.15	56.0110	55.9263	0.15
23	24	58.6820	58.7892	-0.18	58.6820	58.7892	-0.18
24	24	61.3550	61.3038	0.08	61.3550	61.3799	-0.04
25	24	64.0240	64.0148	0.01	63.0410	62.9103	0.21
26	24	66.6960	66.7323	-0.05	65.3420	65.4490	-0.16
27	24	69.3700	69.4882	-0.17	67.8040	67.7023	0.15
28	24	72.0470	71.9808	0.09	70.3110	70.2451	0.09
29	24	74.7250	74.8684	-0.19	72.8420	72.7569	0.12

		AMSCRA WITH COL.RESOL.			AMSCRA WITH SEP.RESOL.		
n	b	analy.	sim.	error %	analy.	sim.	error %
30	24	77.4040	77.5132	-0.14	75.3940	75.3869	0.01
2	25	4.0000	3.9656	0.86	4.0000	3.9656	0.86
3	25	6.3750	6.3903	-0.24	6.3750	6.3903	-0.24
4	25	8.8240	8.7947	0.33	8.8240	8.7947	0.33
5	25	11.3260	11.3235	0.02	11.3260	11.3235	0.02
6	25	13.8650	13.8289	0.26	13.8650	13.8289	0.26
7	25	16.4290	16.4241	0.03	16.4290	16.4241	0.03
8	25	19.0120	19.0245	-0.07	19.0120	19.0245	-0.07
9	25	21.6100	21.6337	-0.11	21.6100	21.6337	-0.11
10	25	24.2200	24.1600	0.25	24.2200	24.1600	0.25
11	25	26.8390	26.9067	-0.25	26.8390	26.9067	-0.25
12	25	29.4670	29.5222	-0.19	29.4670	29.5222	-0.19
13	25	32.1020	32.1407	-0.12	32.1020	32.1407	-0.12
14	25	34.7430	34.7447	0.00	34.7430	34.7447	0.00
15	25	37.3890	37.4433	-0.15	37.3890	37.4433	-0.15
16	25	40.0390	39.9694	0.17	40.0390	39.9694	0.17
17	25	42.6930	42.6520	0.10	42.6930	42.6520	0.10
18	25	45.3510	45.3124	0.09	45.3510	45.3124	0.09
19	25	48.0120	47.9503	0.13	48.0120	47.9503	0.13
20	25	50.6760	50.7586	-0.16	50.6760	50.7586	-0.16
21	25	53.3430	53.2577	0.16	53.3430	53.2577	0.16
22	25	56.0110	55.9457	0.12	56.0110	55.9457	0.12
23	25	58.6820	58.7609	-0.13	58.6820	58.7609	-0.13
24	25	61.3550	61.4551	-0.16	61.3550	61.4551	-0.16
25	25	64.0300	64.1019	-0.11	64.0300	64.1203	-0.14
26	25	66.7010	66.6695	0.05	65.7150	65.5291	0.28
27	25	69.3750	69.3695	0.01	68.0160	68.1507	-0.20
28	25	72.0500	72.1985	-0.21	70.4770	70.4244	0.07
29	25	74.7280	74.6080	0.16	72.9830	72.7699	0.29
30	25	77.4070	77.5095	-0.13	75.5140	75.5709	-0.08
2	26	4.0000	4.0054	-0.13	4.0000	4.0054	-0.13
3	26	6.3750	6.3814	-0.10	6.3750	6.3814	-0.10
4	26	8.8240	8.8223	0.02	8.8240	8.8223	0.02
5	26	11.3260	11.3025	0.21	11.3260	11.3025	0.21
6	26	13.8650	13.8561	0.06	13.8650	13.8561	0.06
7	26	16.4290	16.4041	0.15	16.4290	16.4041	0.15
8	26	19.0120	18.9905	0.11	19.0120	18.9905	0.11
9	26	21.6100	21.6203	-0.05	21.6100	21.6203	-0.05
10	26	24.2200	24.1779	0.17	24.2200	24.1779	0.17
11	26	26.8390	26.8736	-0.13	26.8390	26.8736	-0.13
12	26	29.4670	29.4013	0.22	29.4670	29.4013	0.22
13	26	32.1020	32.0865	0.05	32.1020	32.0865	0.05
14	26	34.7430	34.7075	0.10	34.7430	34.7075	0.10
15	26	37.3890	37.4628	-0.20	37.3890	37.4628	-0.20
16	26	40.0390	40.0466	-0.02	40.0390	40.0466	-0.02
17	26	42.6930	42.6713	0.05	42.6930	42.6713	0.05
18	26	45.3510	45.2703	0.18	45.3510	45.2703	0.18
19	26	48.0120	48.0098	0.00	48.0120	48.0098	0.00
20	26	50.6760	50.7951	-0.24	50.6760	50.7951	-0.24
21	26	53.3430	53.3852	-0.08	53.3430	53.3852	-0.08
22	26	56.0110	56.1066	-0.17	56.0110	56.1066	-0.17
23	26	58.6820	58.6835	0.00	58.6820	58.6835	0.00
24	26	61.3550	61.2484	0.17	61.3550	61.2484	0.17
25	26	64.0300	63.9501	0.12	64.0300	63.9501	0.12
26	26	66.7070	66.7245	-0.03	66.7070	66.7367	-0.04
27	26	69.3790	69.3325	0.07	68.3910	68.2383	0.22
28	26	72.0540	72.0768	-0.03	70.6910	70.8544	-0.23
29	26	74.7320	74.7312	0.00	73.1510	73.2220	-0.10
30	26	77.4110	77.3821	0.04	75.6560	75.5614	0.13
2	27	4.0000	3.9704	0.74	4.0000	3.9704	0.74
3	27	6.3750	6.4061	-0.49	6.3750	6.4061	-0.49
4	27	8.8240	8.8547	-0.35	8.8240	8.8547	-0.35
5	27	11.3260	11.3493	-0.21	11.3260	11.3493	-0.21
6	27	13.8650	13.8579	0.05	13.8650	13.8579	0.05

		AMSCRA WITH COL.RESOL.			AMSCRA WITH SEP.RESOL.		
n	b	analy.	sim.	error %	analy.	sim.	error %
7	27	16.4290	16.4280	0.01	16.4290	16.4280	0.01
8	27	19.0120	19.0045	0.04	19.0120	19.0045	0.04
9	27	21.6100	21.6239	-0.06	21.6100	21.6239	-0.06
10	27	24.2200	24.2821	-0.26	24.2200	24.2821	-0.26
11	27	26.8390	26.7470	0.34	26.8390	26.7470	0.34
12	27	29.4670	29.4661	0.00	29.4670	29.4661	0.00
13	27	32.1020	32.0549	0.15	32.1020	32.0549	0.15
14	27	34.7430	34.6801	0.18	34.7430	34.6801	0.18
15	27	37.3890	37.4015	-0.03	37.3890	37.4015	-0.03
16	27	40.0390	40.1038	-0.16	40.0390	40.1038	-0.16
17	27	42.6930	42.6921	0.00	42.6930	42.6921	0.00
18	27	45.3510	45.4244	-0.16	45.3510	45.4244	-0.16
19	27	48.0120	48.0187	-0.01	48.0120	48.0187	-0.01
20	27	50.6760	50.6413	0.07	50.6760	50.6413	0.07
21	27	53.3430	53.2595	0.16	53.3430	53.2595	0.16
22	27	56.0110	56.0909	-0.14	56.0110	56.0909	-0.14
23	27	58.6820	58.6615	0.03	58.6820	58.6615	0.03
24	27	61.3550	61.1693	0.30	61.3550	61.1693	0.30
25	27	64.0300	64.2094	-0.28	64.0300	64.2094	-0.28
26	27	66.7070	66.5915	0.17	66.7070	66.5915	0.17
27	27	69.3850	69.2884	0.14	69.3850	69.4381	-0.08
28	27	72.0590	72.2431	-0.26	71.0680	70.9957	0.10
29	27	74.7350	74.6687	0.09	73.3680	73.2219	0.20
30	27	77.4140	77.5065	-0.12	75.8270	75.7976	0.04
2	28	4.0000	4.0050	-0.13	4.0000	4.0050	-0.13
3	28	6.3750	6.3671	0.12	6.3750	6.3671	0.12
4	28	8.8240	8.8809	-0.64	8.8240	8.8809	-0.64
5	28	11.3260	11.3303	-0.04	11.3260	11.3303	-0.04
6	28	13.8650	13.8814	-0.12	13.8650	13.8814	-0.12
7	28	16.4290	16.4469	-0.11	16.4290	16.4469	-0.11
8	28	19.0120	19.0281	-0.08	19.0120	19.0281	-0.08
9	28	21.6100	21.6631	-0.25	21.6100	21.6631	-0.25
10	28	24.2200	24.2207	0.00	24.2200	24.2207	0.00
11	28	26.8390	26.8081	0.12	26.8390	26.8081	0.12
12	28	29.4670	29.4693	-0.01	29.4670	29.4693	-0.01
13	28	32.1020	32.0447	0.18	32.1020	32.0447	0.18
14	28	34.7430	34.6777	0.19	34.7430	34.6777	0.19
15	28	37.3890	37.3750	0.04	37.3890	37.3750	0.04
16	28	40.0390	39.9392	0.25	40.0390	39.9392	0.25
17	28	42.6930	42.6758	0.04	42.6930	42.6758	0.04
18	28	45.3510	45.3477	0.01	45.3510	45.3477	0.01
19	28	48.0120	47.9529	0.12	48.0120	47.9529	0.12
20	28	50.6760	50.6190	0.11	50.6760	50.6190	0.11
21	28	53.3430	53.2858	0.11	53.3430	53.2858	0.11
22	28	56.0110	56.1303	-0.21	56.0110	56.1303	-0.21
23	28	58.6820	58.6901	-0.01	58.6820	58.6901	-0.01
24	28	61.3550	61.3287	0.04	61.3550	61.3287	0.04
25	28	64.0300	63.8161	0.33	64.0300	63.8161	0.33
26	28	66.7070	66.6520	0.08	66.7070	66.6520	0.08
27	28	69.3850	69.4542	-0.10	69.3850	69.4542	-0.10
28	28	72.0640	72.0177	0.06	72.0640	72.1174	-0.07
29	28	74.7400	74.8778	-0.18	73.7470	73.7210	0.04
30	28	77.4180	77.3642	0.07	76.0450	76.0197	0.03
2	29	4.0000	3.9838	0.40	4.0000	3.9838	0.40
3	29	6.3750	6.3598	0.24	6.3750	6.3598	0.24
4	29	8.8240	8.8290	-0.06	8.8240	8.8290	-0.06
5	29	11.3260	11.2588	0.59	11.3260	11.2588	0.59
6	29	13.8650	13.8634	0.01	13.8650	13.8634	0.01
7	29	16.4290	16.4279	0.01	16.4290	16.4279	0.01
8	29	19.0120	18.9781	0.18	19.0120	18.9781	0.18
9	29	21.6100	21.5402	0.32	21.6100	21.5402	0.32
10	29	24.2200	24.1936	0.11	24.2200	24.1936	0.11
11	29	26.8390	26.8187	0.08	26.8390	26.8187	0.08
12	29	29.4670	29.5703	-0.35	29.4670	29.5703	-0.35

		AMSCRA WITH COL.RESOL.			AMSCRA WITH SEP.RESOL.		
n	b	analy.	sim.	error %	analy.	sim.	error %
13	29	32.1020	32.0530	0.15	32.1020	32.0530	0.15
14	29	34.7430	34.5781	0.47	34.7430	34.5781	0.47
15	29	37.3890	37.3948	-0.02	37.3890	37.3948	-0.02
16	29	40.0390	40.0455	-0.02	40.0390	40.0455	-0.02
17	29	42.6930	42.7518	-0.14	42.6930	42.7518	-0.14
18	29	45.3510	45.4305	-0.18	45.3510	45.4305	-0.18
19	29	48.0120	48.0296	-0.04	48.0120	48.0296	-0.04
20	29	50.6760	50.5929	0.16	50.6760	50.5929	0.16
21	29	53.3430	53.4695	-0.24	53.3430	53.4695	-0.24
22	29	56.0110	55.9745	0.07	56.0110	55.9745	0.07
23	29	58.6820	58.6523	0.05	58.6820	58.6523	0.05
24	29	61.3550	61.3703	-0.02	61.3550	61.3703	-0.02
25	29	64.0300	64.0935	-0.10	64.0300	64.0935	-0.10
26	29	66.7070	66.6816	0.04	66.7070	66.6816	0.04
27	29	69.3850	69.4470	-0.09	69.3850	69.4470	-0.09
28	29	72.0640	72.0811	-0.02	72.0640	72.0811	-0.02
29	29	74.7450	74.7899	-0.06	74.7450	74.7423	0.00
30	29	77.4220	77.5541	-0.17	76.4280	76.6523	-0.29
2	30	4.0000	3.9358	1.60	4.0000	3.9358	1.60
3	30	6.3750	6.3694	0.09	6.3750	6.3694	0.09
4	30	8.8240	8.7997	0.28	8.8240	8.7997	0.28
5	30	11.3260	11.3307	-0.04	11.3260	11.3307	-0.04
6	30	13.8650	13.8669	-0.01	13.8650	13.8669	-0.01
7	30	16.4290	16.5016	-0.44	16.4290	16.5016	-0.44
8	30	19.0120	19.0155	-0.02	19.0120	19.0155	-0.02
9	30	21.6100	21.5635	0.22	21.6100	21.5635	0.22
10	30	24.2200	24.1942	0.11	24.2200	24.1942	0.11
11	30	26.8390	26.8338	0.02	26.8390	26.8338	0.02
12	30	29.4670	29.5311	-0.22	29.4670	29.5311	-0.22
13	30	32.1020	32.0968	0.02	32.1020	32.0968	0.02
14	30	34.7430	34.7971	-0.16	34.7430	34.7971	-0.16
15	30	37.3890	37.4008	-0.03	37.3890	37.4008	-0.03
16	30	40.0390	40.0770	-0.09	40.0390	40.0770	-0.09
17	30	42.6930	42.7618	-0.16	42.6930	42.7618	-0.16
18	30	45.3510	45.3885	-0.08	45.3510	45.3885	-0.08
19	30	48.0120	48.0303	-0.04	48.0120	48.0303	-0.04
20	30	50.6760	50.5428	0.26	50.6760	50.5428	0.26
21	30	53.3430	53.3414	0.00	53.3430	53.3414	0.00
22	30	56.0110	55.9346	0.14	56.0110	55.9346	0.14
23	30	58.6820	58.7024	-0.03	58.6820	58.7024	-0.03
24	30	61.3550	61.3763	-0.03	61.3550	61.3763	-0.03
25	30	64.0300	63.9523	0.12	64.0300	63.9523	0.12
26	30	66.7070	66.6325	0.11	66.7070	66.6325	0.11
27	30	69.3850	69.2812	0.15	69.3850	69.2812	0.15
28	30	72.0640	72.1373	-0.10	72.0640	72.1373	-0.10
29	30	74.7450	74.6784	0.09	74.7450	74.6784	0.09
30	30	77.4270	77.4130	0.02	77.4270	77.4542	-0.04

D. COMPARISON BETWEEEN SEPARATE AND COLLECTIVE RESOLUTION

If $n < b$, there is no difference between two methods. If $n \geq b$ AMSCRA with separated resolution gives a better result than with collective resolution. Comparison between the two methods is given in Figures 3.13. 3.14, 3.15 and 3.16 for various values of b. From these figures it can be seen that AMSCRA with separated resolution gives better result. These results are also provided in Table III. From Table III it can be seen that as n increases, the advantage of separate resolution over collective resolution also increases.

TABLE III

COMPARISON BETWEEN TWO VERSIONS OF THE AMSCRA.

N	B	COLLECTIVE AMSCRA	SEPARATE AMSCRA	VARIATION %
2	1	3.0000	3.0000	0.0000
3	1	5.2500	4.8330	7.9429
4	1	7.6200	6.7350	11.6142
5	1	10.0610	8.6510	14.0145
6	1	12.5500	10.5780	15.7131
7	1	15.0710	12.5130	16.9730
8	1	17.6180	14.4550	17.9532
9	1	20.1840	16.4010	18.7426
10	1	22.7650	18.3510	19.3894
11	1	25.3580	20.3040	19.9306
12	1	27.9630	22.2600	20.3948
13	1	30.5760	24.2180	20.7941
14	1	33.1960	26.1770	21.1441
15	1	35.8230	28.1390	21.4499
16	1	38.4560	30.1020	21.7235
17	1	41.0940	32.0660	21.9691
18	1	43.7360	34.0310	22.1900
19	1	46.3830	35.9980	22.3897
20	1	49.0330	37.9650	22.5726
21	1	51.6860	39.9330	22.7392
22	1	54.3420	41.9010	22.8939
23	1	57.0010	43.8710	23.0347
24	1	59.6630	45.8410	23.1668
25	1	62.3260	47.8120	23.2872
26	1	64.9920	49.7830	23.4013
27	1	67.6600	51.7540	23.5087
28	1	70.3290	53.7260	23.6076
29	1	73.0010	55.6990	23.7010
30	1	75.6730	57.6720	23.7879
2	2	4.0000	4.0000	0.0000
3	2	6.0000	5.7500	4.1667
4	2	8.4760	7.9740	5.9226
5	2	10.9080	10.1820	6.6557
6	2	13.4110	12.3700	7.7623
7	2	15.9370	14.5730	8.5587
8	2	18.4880	16.7850	9.2114
9	2	21.0580	19.0000	9.7730
10	2	23.6420	21.2200	10.2445
11	2	26.2380	23.4430	10.6525
12	2	28.8440	25.6680	11.0110
13	2	31.4590	27.8970	11.3227
14	2	34.0810	30.1270	11.6018
15	2	36.7090	32.3590	11.8500
16	2	39.3430	34.5920	12.0758
17	2	41.9820	36.8270	12.2791
18	2	44.6250	39.0630	12.4639
19	2	47.2720	41.3000	12.6333
20	2	49.9230	43.5390	12.7877
21	2	52.5770	45.7780	12.9315
22	2	55.2330	48.0180	13.0628
23	2	57.8930	50.2580	13.1881
24	2	60.5550	52.5000	13.3020
25	2	63.2190	54.7420	13.4089
26	2	65.8850	56.9850	13.5084
27	2	68.5530	59.2280	13.6026
28	2	71.2230	61.4720	13.6908
29	2	73.8940	63.7160	13.7738
30	2	76.5680	65.9600	13.8544

N	B	COLLECTIVE AMSCRA	SEPARATE AMSCRA	VARIATION %
3	3	6.3750	6.3750	0.0000
4	3	8.6690	8.1280	6.2406
5	3	11.1850	10.4570	6.5087
6	3	13.6780	12.7780	6.5799
7	3	16.2130	15.0110	7.4138
8	3	18.7690	17.2480	8.1038
9	3	21.3410	19.5090	8.5844
10	3	23.9280	21.7770	8.9895
11	3	26.5260	24.0450	9.3531
12	3	29.1340	26.3150	9.6760
13	3	31.7510	28.5870	9.9650
14	3	34.3740	30.8620	10.2170
15	3	37.0030	33.1390	10.4424
16	3	39.6380	35.4180	10.6463
17	3	42.2780	37.6970	10.8354
18	3	44.9220	39.9790	11.0035
19	3	47.5700	42.2610	11.1604
20	3	50.2210	44.5440	11.3040
21	3	52.8760	46.8290	11.4362
22	3	55.5330	49.1140	11.5589
23	3	58.1930	51.4000	11.6732
24	3	60.8550	53.6870	11.7788
25	3	63.5200	55.9740	11.8797
26	3	66.1870	58.2630	11.9721
27	3	68.8550	60.5510	12.0601
28	3	71.5250	62.8410	12.1412
29	3	74.1970	65.1300	12.2202
30	3	76.8700	67.4210	12.2922
4	4	8.8240	8.8240	0.0000
5	4	11.2500	10.5870	5.8933
6	4	13.7880	12.9460	6.1068
7	4	16.3220	15.3710	5.8265
8	4	18.8810	17.6940	6.2867
9	4	21.4580	19.9600	6.9811
10	4	24.0470	22.2420	7.5061
11	4	26.6480	24.5550	7.8542
12	4	29.2580	26.8760	8.1414
13	4	31.8760	29.1930	8.4170
14	4	34.5000	31.5080	8.6725
15	4	37.1310	33.8240	8.9063
16	4	39.7670	36.1440	9.1106
17	4	42.4080	38.4650	9.2978
18	4	45.0530	40.7880	9.4666
19	4	47.7010	43.1120	9.6203
20	4	50.3530	45.4360	9.7651
21	4	53.0080	47.7620	9.8966
22	4	55.6660	50.0880	10.0205
23	4	58.3260	52.4160	10.1327
24	4	60.9890	54.7440	10.2396
25	4	63.6540	57.0720	10.3403
26	4	66.3210	59.4020	10.4326
27	4	68.9900	61.7320	10.5204
28	4	71.6600	64.0620	10.6028
29	4	74.3330	66.3930	10.6817
30	4	77.0060	68.7250	10.7537
5	5	11.3260	11.3260	0.0000
6	5	13.8210	13.0730	5.4121
7	5	16.3770	15.4300	5.7825
8	5	18.9390	17.9070	5.4491
9	5	21.5180	20.3250	5.5442
10	5	24.1100	22.6560	6.0307
11	5	26.7130	24.9480	6.6073
12	5	29.3250	27.2610	7.0384
13	5	31.9440	29.6080	7.3128
14	5	34.5700	31.9690	7.5239
15	5	37.2020	34.3270	7.7281

N	B	COLLECTIVE AMSCRA	SEPARATE AMSCRA	VARIATION %
16	5	39.8390	36.6760	7.9395
17	5	42.4800	39.0220	8.1403
18	5	45.1260	41.3710	8.3211
19	5	47.7750	43.7240	8.4793
20	5	50.4280	46.0800	8.6222
21	5	53.0830	48.4360	8.7542
22	5	55.7420	50.7920	8.8802
23	5	58.4030	53.1480	8.9978
24	5	61.0660	55.5050	9.1065
25	5	63.7310	57.8640	9.2059
26	5	66.3990	60.2220	9.3029
27	5	69.0680	62.5820	9.3907
28	5	71.7390	64.9420	9.4746
29	5	74.4110	67.3020	9.5537
30	5	77.0850	69.6630	9.6283
6	6	13.8650	13.8650	0.0000
7	6	16.3990	15.6020	4.8601
8	6	18.9720	17.9520	5.3763
9	6	21.5550	20.4430	5.1589
10	6	24.1490	22.9240	5.0727
11	6	26.7530	25.3420	5.2742
12	6	29.3670	27.6900	5.7105
13	6	31.9880	30.0080	6.1898
14	6	34.6150	32.3440	6.5607
15	6	37.2480	34.7160	6.7977
16	6	39.8860	37.1100	6.9598
17	6	42.5280	39.5050	7.1083
18	6	45.1750	41.8900	7.2717
19	6	47.8250	44.2650	7.4438
20	6	50.4780	46.6380	7.6073
21	6	53.1340	49.0140	7.7540
22	6	55.7930	51.3960	7.8809
23	6	58.4540	53.7810	7.9943
24	6	61.1180	56.1660	8.1024
25	6	63.7840	58.5510	8.2043
26	6	66.4510	60.9350	8.3009
27	6	69.1210	63.3190	8.3940
28	6	71.7920	65.7050	8.4787
29	6	74.4650	68.0910	8.5597
30	6	77.1390	70.4770	8.6364
7	7	16.4290	16.4290	0.0000
8	7	18.9890	18.1550	4.3920
9	7	21.5770	20.4990	4.9961
10	7	24.1750	22.9910	4.8976
11	7	26.7810	25.4980	4.7907
12	7	29.3960	27.9810	4.8136
13	7	32.0180	30.4080	5.0284
14	7	34.6470	32.7740	5.4060
15	7	37.2810	35.1120	5.8180
16	7	39.9200	37.4660	6.1473
17	7	42.5630	39.8550	6.3623
18	7	45.2100	42.2730	6.4964
19	7	47.8610	44.7000	6.6045
20	7	50.5150	47.1190	6.7228
21	7	53.1720	49.5240	6.8608
22	7	55.8310	51.9200	7.0051
23	7	58.4930	54.3150	7.1427
24	7	61.1570	56.7140	7.2649
25	7	63.8230	59.1190	7.3704
26	7	66.4910	61.5280	7.4642
27	7	69.1610	63.9390	7.5505
28	7	71.8330	66.3490	7.6344
29	7	74.5060	68.7570	7.7162
30	7	77.1800	71.1640	7.7948
8	8	19.0120	19.0120	0.0000
9	8	21.5910	20.7310	3.9831

N	B	COLLECTIVE AMSCRA	SEPARATE AMSCRA	VARIATION %
10	8	24.1910	23.0690	4.6381
11	8	26.8010	25.5580	4.6379
12	8	29.4180	28.0740	4.5686
13	8	32.0410	30.5900	4.5286
14	8	34.6710	33.0810	4.5860
15	8	37.3060	35.5200	4.7874
16	8	39.9460	37.9020	5.1169
17	8	42.5900	40.2580	5.4755
18	8	45.2380	42.6250	5.7761
19	8	47.8890	45.0260	5.9784
20	8	50.5440	47.4610	6.0996
21	8	53.2010	49.9120	6.1822
22	8	55.8610	52.3630	6.2620
23	8	58.5240	54.8000	6.3632
24	8	61.1880	57.2240	6.4784
25	8	63.8550	59.6380	6.6040
26	8	66.5230	62.0500	6.7240
27	8	69.1930	64.4680	6.8287
28	8	71.8650	66.8920	6.9199
29	8	74.5390	69.3220	6.9990
30	8	77.2130	71.7550	7.0688
9	9	21.6100	21.6100	0.0000
10	9	24.2040	23.3230	3.6399
11	9	26.8140	25.6560	4.3186
12	9	29.4340	28.1420	4.3895
13	9	32.0590	30.6590	4.3669
14	9	34.6900	33.1890	4.3269
15	9	37.3260	35.7170	4.3107
16	9	39.9670	38.2190	4.3736
17	9	42.6120	40.6690	4.5597
18	9	45.2600	43.0650	4.8498
19	9	47.9120	45.4350	5.1699
20	9	50.5670	47.8140	5.4443
21	9	53.2250	50.2230	5.6402
22	9	55.8860	52.6690	5.7564
23	9	58.5490	55.1380	5.8259
24	9	61.2130	57.6130	5.8811
25	9	63.8800	60.0810	5.9471
26	9	66.5490	62.5340	6.0331
27	9	69.2200	64.9730	6.1355
28	9	71.8920	67.4020	6.2455
29	9	74.5660	69.8300	6.3514
30	9	77.2410	72.2620	6.4461
10	10	24.2200	24.2200	0.0000
11	10	26.8250	25.9280	3.3439
12	10	29.4440	28.2570	4.0314
13	10	32.0720	30.7400	4.1532
14	10	34.7050	33.2570	4.1723
15	10	37.3420	35.7910	4.1535
16	10	39.9830	38.3360	4.1193
17	10	42.6290	40.8780	4.1075
18	10	45.2780	43.3880	4.1742
19	10	47.9310	45.8480	4.3458
20	10	50.5870	48.2570	4.6059
21	10	53.2450	50.6390	4.8944
22	10	55.9060	53.0270	5.1497
23	10	58.5690	55.4430	5.3373
24	10	61.2350	57.8950	5.4544
25	10	63.9020	60.3760	5.5178
26	10	66.5710	62.8710	5.5580
27	10	69.2420	65.3640	5.6006
28	10	71.9150	67.8470	5.6567
29	10	74.5890	70.3130	5.7327
30	10	77.2640	72.7640	5.8242
11	11	26.8390	26.8390	0.0000
12	11	29.4550	28.5440	3.0929

N	B	COLLECTIVE AMSCRA	SEPARATE AMSCRA	VARIATION %
13	11	32.0810	30.8700	3.7748
14	11	34.7150	33.3500	3.9320
15	11	37.3540	35.8660	3.9835
16	11	39.9970	38.4010	3.9903
17	11	42.6430	40.9530	3.9631
18	11	45.2930	43.5140	3.9278
19	11	47.9470	46.0660	3.9231
20	11	50.6030	48.5840	3.9899
21	11	53.2620	51.0520	4.1493
22	11	55.9230	53.4710	4.3846
23	11	58.5870	55.8640	4.6478
24	11	61.2530	58.2600	4.8863
25	11	63.9210	60.6820	5.0672
26	11	66.5900	63.1380	5.1840
27	11	69.2610	65.6260	5.2483
28	11	71.9340	68.1340	5.2826
29	11	74.6080	70.6480	5.3077
30	11	77.2840	73.1560	5.3413
12	12	29.4670	29.4670	0.0000
13	12	32.0900	31.1690	2.8701
14	12	34.7230	33.4910	3.5481
15	12	37.3630	35.9690	3.7310
16	12	40.0080	38.4850	3.8067
17	12	42.6550	41.0200	3.8331
18	12	45.3060	43.5730	3.8251
19	12	47.9600	46.1430	3.7886
20	12	50.6160	48.7180	3.7498
21	12	53.2760	51.2780	3.7503
22	12	55.9380	53.8020	3.8185
23	12	58.6020	56.2770	3.9674
24	12	61.2680	58.7050	4.1833
25	12	63.9360	61.1070	4.4247
26	12	66.6060	63.5110	4.6467
27	12	69.2780	65.9370	4.8226
28	12	71.9510	68.3960	4.9409
29	12	74.6250	70.8890	5.0064
30	12	77.3010	73.4050	5.0400
13	13	32.1020	32.1020	0.0000
14	13	34.7320	33.8010	2.6805
15	13	37.3710	36.1200	3.3475
16	13	40.0150	38.5970	3.5437
17	13	42.6650	41.1120	3.6400
18	13	45.3170	43.6460	3.6874
19	13	47.9710	46.2000	3.6918
20	13	50.6280	48.7730	3.6640
21	13	53.2880	51.3580	3.6218
22	13	55.9500	53.9430	3.5871
23	13	58.6150	56.5090	3.5929
24	13	61.2820	59.0380	3.6618
25	13	63.9500	61.5190	3.8014
26	13	66.6200	63.9550	4.0003
27	13	69.2920	66.3650	4.2242
28	13	71.9650	68.7760	4.4313
29	13	74.6400	71.2060	4.6008
30	13	77.3160	73.6680	4.7183
14	14	34.7430	34.7430	0.0000
15	14	37.3790	36.4390	2.5148
16	14	40.0220	38.7560	3.1633
17	14	42.6710	41.2310	3.3747
18	14	45.3240	43.7450	3.4838
19	14	47.9800	46.2790	3.5452
20	14	50.6380	48.8340	3.5625
21	14	53.2990	51.4070	3.5498
22	14	55.9610	53.9960	3.5114
23	14	58.6260	56.5930	3.4677
24	14	61.2930	59.1860	3.4376

N	B	COLLECTIVE AMSCRA	SEPARATE AMSCRA	VARIATION %
25	14	63.9620	61.7570	3.4474
26	14	66.6330	64.2900	3.5163
27	14	69.3050	66.7750	3.6505
28	14	71.9780	69.2180	3.8345
29	14	74.6530	71.6370	4.0400
30	14	77.3300	74.0530	4.2377
15	15	37.3890	37.3890	0.0000
16	15	40.0300	39.0830	2.3657
17	15	42.6780	41.3980	2.9992
18	15	45.3300	43.8710	3.2186
19	15	47.9870	46.3840	3.3405
20	15	50.6460	48.9180	3.4119
21	15	53.3080	51.4730	3.4423
22	15	55.9710	54.0460	3.4393
23	15	58.6360	56.6370	3.4092
24	15	61.3030	59.2390	3.3669
25	15	63.9720	61.8450	3.3249
26	15	66.6430	64.4440	3.2997
27	15	69.3160	67.0180	3.3153
28	15	71.9900	69.5540	3.3838
29	15	74.6650	72.0440	3.5103
30	15	77.3420	74.4940	3.6823
16	16	40.0390	40.0390	0.0000
17	16	42.6850	41.7320	2.2326
18	16	45.3360	44.0450	2.8476
19	16	47.9930	46.5160	3.0775
20	16	50.6520	49.0280	3.2062
21	16	53.3150	51.5620	3.2880
22	16	55.9790	54.1170	3.3262
23	16	58.6450	56.6900	3.3336
24	16	61.3120	59.2810	3.3126
25	16	63.9810	61.8850	3.2760
26	16	66.6520	64.4980	3.2317
27	16	69.3250	67.1110	3.1937
28	16	72.0000	69.7140	3.1750
29	16	74.6760	72.2910	3.1938
30	16	77.3520	74.8300	3.2604
17	17	42.6930	42.6930	0.0000
18	17	45.3430	44.3850	2.1128
19	17	47.9980	46.6960	2.7126
20	17	50.6570	49.1650	2.9453
21	17	53.3200	51.6770	3.0814
22	17	55.9850	54.2110	3.1687
23	17	58.6520	56.7650	3.2173
24	17	61.3200	59.3380	3.2322
25	17	63.9900	61.9290	3.2208
26	17	66.6610	64.5340	3.1908
27	17	69.3340	67.1490	3.1514
28	17	72.0080	69.7700	3.1080
29	17	74.6850	72.3900	3.0729
30	17	77.3620	74.9960	3.0583
18	18	45.3510	45.3510	0.0000
19	18	48.0050	47.0410	2.0081
20	18	50.6630	49.3510	2.5897
21	18	53.3250	51.8190	2.8242
22	18	55.9900	54.3300	2.9648
23	18	58.6570	56.8630	3.0585
24	18	61.3270	59.4170	3.1145
25	18	63.9970	61.9910	3.1345
26	18	66.6690	64.5810	3.1319
27	18	69.3420	67.1860	3.1092
28	18	72.0160	69.8020	3.0743
29	18	74.6920	72.4270	3.0325
30	18	77.3700	75.0550	2.9921
19	19	48.0120	48.0120	0.0000
20	19	50.6690	49.7010	1.9104

N	B	COLLECTIVE AMSCRA	SEPARATE AMSCRA	VARIATION %
22	19	55.9940	54.4760	2.7110
21	19	53.3300	52.0090	2.4770
23	19	58.6620	56.9860	2.8570
24	19	61.3320	59.5190	2.9560
25	19	64.0030	62.0730	3.0155
26	19	66.6750	64.6460	3.0431
27	19	69.3490	67.2370	3.0455
28	19	72.0240	69.8420	3.0295
29	19	74.7000	72.4580	3.0013
30	19	77.3770	75.0840	2.9634
20	20	50.6760	50.6760	0.0000
21	20	53.3360	52.3640	1.8224
22	20	55.9990	54.6710	2.3715
23	20	58.6660	57.1360	2.6080
24	20	61.3360	59.6460	2.7553
25	20	64.0070	62.1780	2.8575
26	20	66.6810	64.7320	2.9229
27	20	69.3550	67.3050	2.9558
28	20	72.0310	69.8950	2.9654
29	20	74.7070	72.5000	2.9542
30	20	77.3840	75.1170	2.9295
21	21	53.3430	53.3430	0.0000
22	21	56.0050	55.0300	1.7409
23	21	58.6710	57.3350	2.2771
24	21	61.3400	59.8000	2.5106
25	21	64.0110	62.3080	2.6605
26	21	66.6850	64.8400	2.7667
27	21	69.3600	67.3940	2.8345
28	21	72.0360	69.9670	2.8722
29	21	74.7130	72.5570	2.8857
30	21	77.3910	75.1620	2.8802
22	22	56.0110	56.0110	0.0000
23	22	58.6760	57.6980	1.6668
24	22	61.3440	60.0020	2.1877
25	22	64.0150	62.4650	2.4213
26	22	66.6880	64.9740	2.5702
27	22	69.3630	67.5050	2.6787
28	22	72.0400	70.0580	2.7512
29	22	74.7180	72.6310	2.7932
30	22	77.3970	75.2210	2.8115
23	23	58.6820	58.6820	0.0000
24	23	61.3490	60.3680	1.5990
25	23	64.0190	62.6710	2.1056
26	23	66.6920	65.1340	2.3361
27	23	69.3670	67.6410	2.4882
28	23	72.0430	70.1720	2.5971
29	23	74.7220	72.7250	2.6726
30	23	77.4010	75.2980	2.7170
24	24	61.3550	61.3550	0.0000
25	24	64.0240	63.0410	1.5354
26	24	66.6960	65.3420	2.0301
27	24	69.3700	67.8040	2.2575
28	24	72.0470	70.3110	2.4095
29	24	74.7250	72.8420	2.5199
30	24	77.4040	75.3940	2.5968
25	25	64.0300	64.0300	0.0000
26	25	66.7010	65.7150	1.4782
27	25	69.3750	68.0160	1.9589
28	25	72.0500	70.4770	2.1832
29	25	74.7280	72.9830	2.3351
30	25	77.4070	75.5140	2.4455
26	26	66.7070	66.7070	0.0000
27	26	69.3790	68.3910	1.4241
28	26	72.0540	70.6910	1.8916
29	26	74.7320	73.1510	2.1156
30	26	77.4110	75.6560	2.2671

N	B	COLLECTIVE AMSCRA	SEPARATE AMSCRA	VARIATION %
27	27	69.3850	69.3850	0.0000
28	27	72.0590	71.0680	1.3753
29	27	74.7350	73.3680	1.8291
30	27	77.4140	75.8270	2.0500
28	28	72.0640	72.0640	0.0000
29	28	74.7400	73.7470	1.3286
30	28	77.4180	76.0450	1.7735
29	29	74.7450	74.7450	0.0000
30	29	77.4220	76.4280	1.2839
30	30	77.4270	77.4270	0.0000

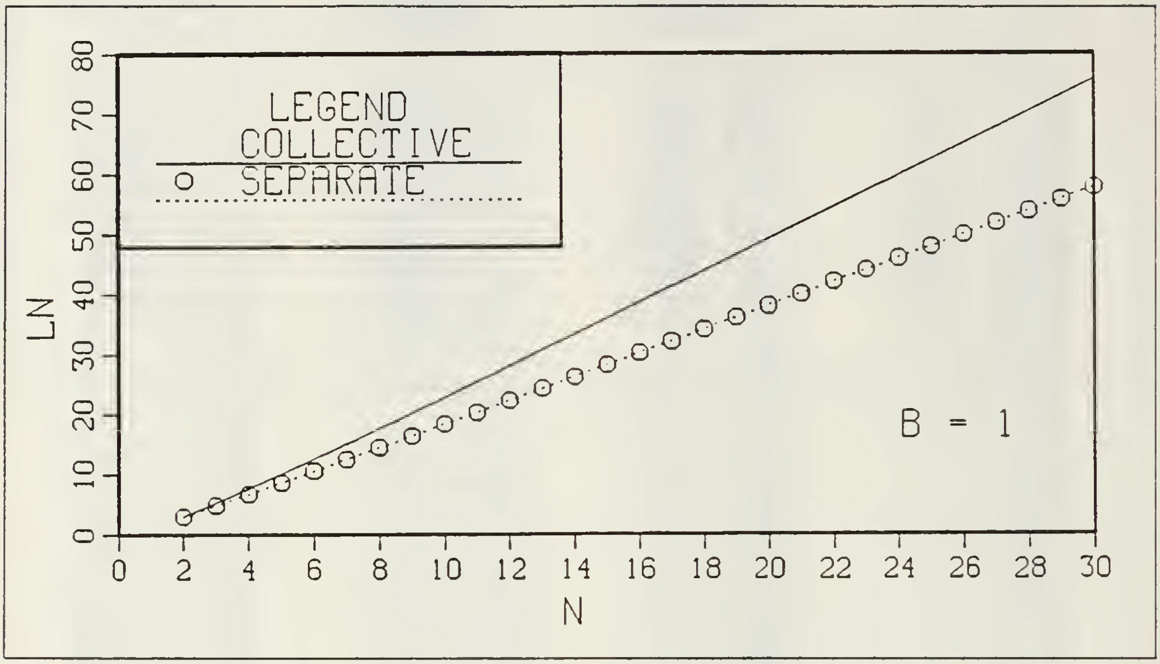

Figure 3.13 Comparison of the Two Versions of the AMSCRA

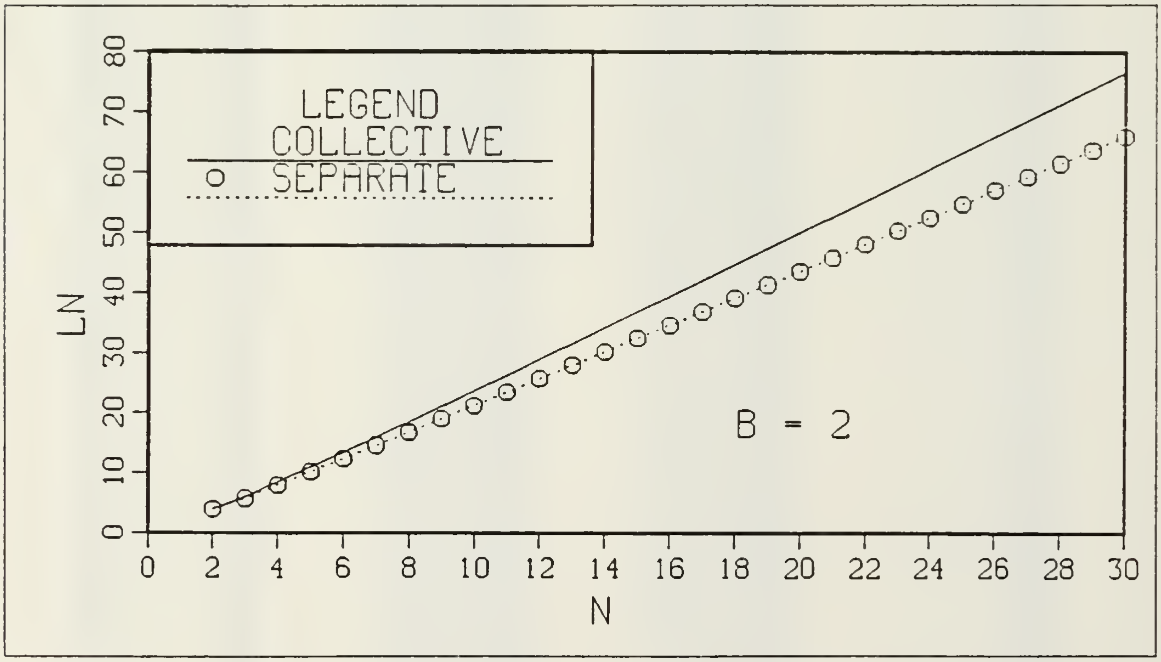

Figure 3.14 Comparison of the Two Versions of the AMSCRA

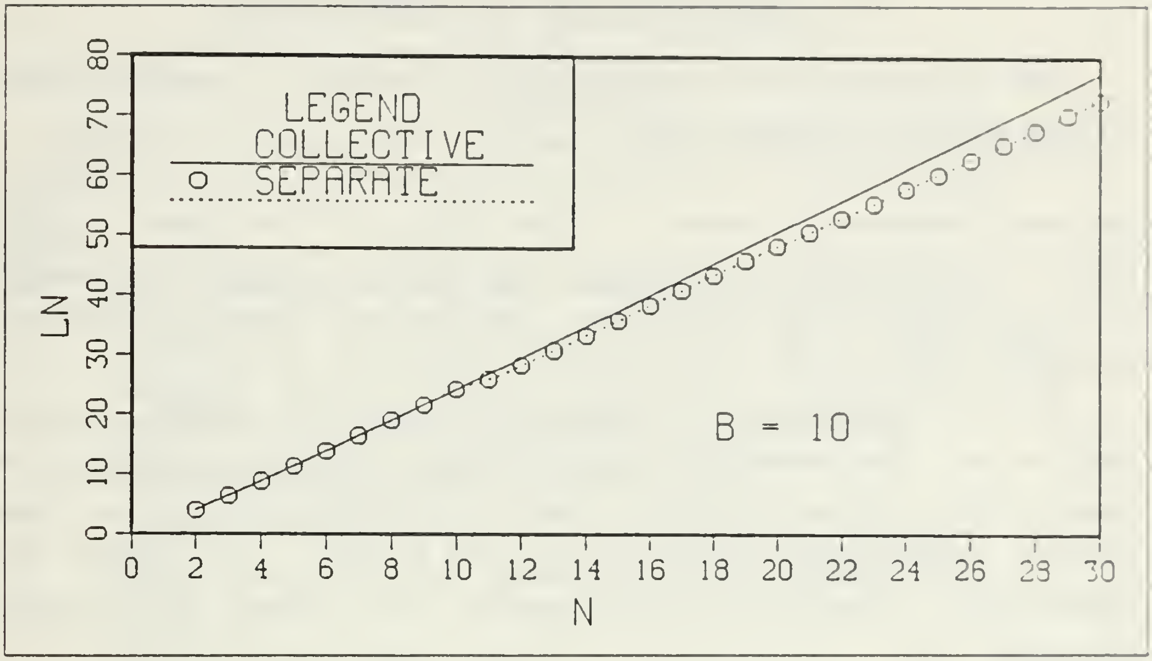

Figure 3.15 Comparison of the Two Versions of the AMSCRA

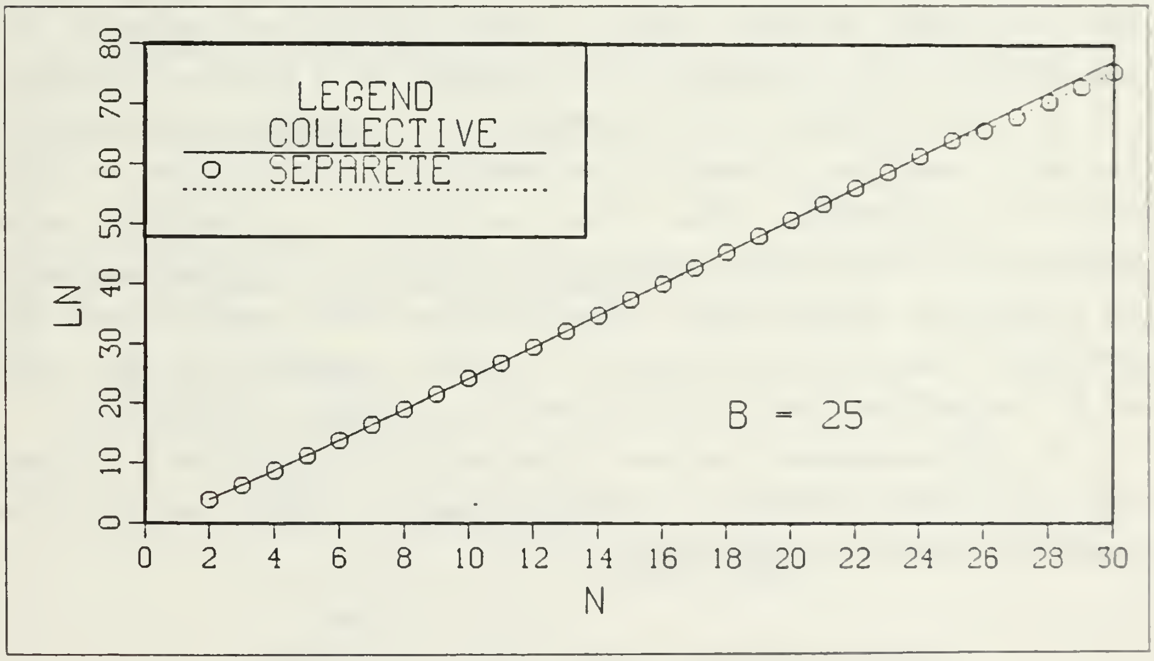

Figure 3.16 Comparison of the Two Versions of the AMSCRA

IV. CONCLUSION AND RECOMMENDATIONS

The principal conclusions of this study on random access to the slotted channel are:

1. The multislot collision resolution algorithm (MSCRA) is the fundamental case of this study. If we open n slots after a collision involving n users, some slots are wasted and therefore the collision resolution interval becomes slightly longer.

2. In general, the adaptive version of MSCRA, called AMSCRA and discussed in two different cases, this adaptive case gives better results than the MSCRA because we used smaller observation intervals in the AMSCRA. (In MSCRA, the observation interval always equals the number of collided messages, n)

3. The first case of the AMSCRA is with collective resolution. at the end of each collision resolution subperiod, we accumulated all collided users collectively and applied the algorithm to this group. Results were better than for the MSCRA, as collisions were resolved in a shorter period.

4. The second case is AMSCRA with separate resolution. In this case the n users involved in collision and divided into two subgroups. The first subgroup is resolved while holding the the second subgroup. Finally the second subgroup is resolved. Using this algorithm the best performance was obtained. As the performance of AMSCRA depends on the value of b, selecting the b=1 provided the optimum performance.

5. The agreement between simulation and analysis is good for our two protocols. These results could therefore be used to calculate throughput.

APPENDIX A

ANALYSIS PROGRAM FOR AMSCRA WITH COLLECTIVE RESOLUTION

```
$JOB
C*** THIS PROGRAM SOLVES THE AMSCRA WITH COLLECTIVE
C     RESOLUTION PROBLEM.STORAGE SHOULD BE AT LEAST 1M.
      INTEGER XX,SS,GG,QQ,KK,JJ,RR,B,I,J,BB,II,Z,G,FF,C
      REAL*8 KL(030,30),PP(030,030),LHAT(030),N
      REAL*8PROB(31,31,31),SUM,MULT,SUB
      REAL*8 A(31,31,31),LHATT,L,M,P,VVV,ATZ,CR,ZS
      REAL*8 JJJ,PPP,LLL,LDENOM,LN,KKK,AAA,ZZ,TT,VV
      PRINT 10
      PRINT 15,'N','B','L(N)'
      KL(1,1)=0.0
      KL(2,1)=0.0
      PRINT 99,0,0,KL(1,1)
      PRINT 99,1,0,KL(2,1)
C
C ***** CALCULATION OF P(N,I)  ***
C
      LHAT(1)=1.0
C********** A(B,J,I) COEFFICIENT ***
C
      DO 25 BB=1,31
         DO 26 JJ=1,31
            II=JJ
            WHILE (II.GE.1) DO
            IF (II.GT.JJ.OR.II.GT.BB) THEN
               A(BB,JJ,II)=0.0
            ELSE IF (II.EQ.JJ.AND.II.EQ.BB) THEN
               FACTI=FACTN(II-1)
               A(BB,JJ,II)=FACTI
            ELSE IF (II.EQ.JJ) THEN
```

```
                 FACTB=FACTN(BB-1)
                 FACTBI=FACTN(BB-II)
                 A(BB,JJ,II)=FACTB/FACTBI
              ELSE
                 N=1
                 ZZ=BB
                 TT=JJ
                 VV=II
                 AAA=((ZZ-VV)**(TT-VV))
                 WHILE (N.LE.(JJ-II)) DO
                    AAA=AAA-A((BB-II+1),(JJ-II+1),N+1)
                    N=N+1
                 END WHILE
                 FACTB=FACTN(BB-1)
                 FACTJ=FACTN(JJ-1)
                 FACTI=FACTN(II-1)
                 FACTBI=FACTN(BB-II)
                 FACTJI=FACTN(JJ-II)
                 A(BB,JJ,II)=(1./FACTI)*FACTB*(1./FACTJI)
     *                      *(1./FACTBI)*FACTJ*AAA
            END IF
            IF (((JJ-1).NE.0).AND.(BB.GE.JJ)) THEN
               CR=JJ
               ATZ=(CR-1.)**(CR-1.)
               PROB(BB,JJ,II)=(1./ATZ)*A(BB,JJ,II)
            END IF
            II=II-1
         END WHILE
   26    CONTINUE
   25    CONTINUE
C
C*****CALCULATION OF L(N) ***
C
         DO 61 K=2,30
         DO 60 B=1,30
```

```
      IF (K.LE.B) THEN
C
C***** MESSAGE NUMBER LESS THAN OR EQUAL SLOT NUMBER***
C***** USE  FIRST FORMULA                          ***
C
         SS=K
         SUM=0.0
         RR=SS-1
         DO 5 I=1,RR
            MULT=PROB(SS+1,SS+1,I+1)*LHAT(SS-I)
            SUM=SUM+MULT
 5       CONTINUE
         ZS=SS
         KL(SS,B)=(1./(1.-PROB(SS+1,SS+1,1)))*(ZS+SUM)
         LHAT(SS)=KL(SS,B)
         PRINT 99,SS,B,KL(SS,B)
      ELSE
C
C******* MESSAGE NUMBER GREATER THAN SLOT NUMBER ***
C******* USE SECOND FORMULA  *****************
C
         M=K
         FACTK=FACTN(K)
         P=B
         L=0.0
         DO 31 J=1,K
            FACTJ=FACTN(J)
            FACTKJ=FACTN(K-J)
            VV=FACTK/(FACTJ*FACTKJ)
            Z=MIN0(B,J)
            DO 30 I=1,Z
               IF ((K-I).EQ.1) THEN
                  LHATT=1.0
               ELSE
                  LHATT=KL(K-I,B)
```

```
                END IF
                L=L+((A(B+1,J+1,I+1)/(P**J))*VV*((P/M)**J)
     *                      *((1.0-(P/M))**(K-J))*LHATT)
 30          CONTINUE
 31       CONTINUE
          SUB=B+L
          LLL=0.0
          G=K+1
          DO 40 J=1,G
             FACTK=FACTN(K)
             FACTJ1=FACTN(J-1)
             FACTJK=FACTN(K-(J-1))
             VVV=FACTK/(FACTJ1*FACTJK)
             PPP=P**(J-1)
             JJJ=(P/M)**(J-1)
             KKK=((1.0-(P/M))**(K-(J-1)))
             LLL=LLL+(A(B+1,J,1)/PPP)*VVV*JJJ*KKK
C            PRINT,LLL
  40      CONTINUE
          LDENOM=1.0/(1.0-LLL)
          LN=LDENOM*SUB
          KL(K,B)=LN
          PRINT 99,K,B,LN
          END IF
 60    CONTINUE
 61    CONTINUE
 15    FORMAT(' ',02X,A1,12X,A1,13X,A4)
 10    FORMAT ('1')
 99    FORMAT(' ',I3,10X,I3,10X,F8.4)
       STOP
       END
C      *************************************************
C      *** FUNCTION SUBPROGRAM TO CALCULATE FACTORIAL N*
C      *************************************************
                    FUNCTION FACTN(N)
```

```
                  FACTN=1.
                  IF (N.GT.1) THEN
                     DO 2 I=2,N
                        FACTN=FACTN*I
 2                   CONTINUE
                  END IF
                  RETURN
                  END
$ENTRY
```

APPENDIX B

SIMULATION FOR AMSCRA WITH COLLECTIVE RESOLUTION

```
C ** THIS PROGRAM WAS WRITTEN TO MAKE THE SIMULATION OF THE
C     AMSCRA WITH COLLECTIVE RESOLUTION.
C *** VARIABLE DEFINITION ***
C K      = INPUT PARAMETER OF DISCRETE UNIFORM DISTRIBUTION
C          THE INTEGERS 1,2,...,K OCCUR WITH EQUAL
C          PROBABILITY. K MUST BE POSITIVE IN GGUD.
C B      =OBSERVATION INTERVAL.
C NR     =INPUT NUMBER OF RANDOM NUMBERS TO BEGENERATED.
C SUM    =DUMMY VARIABLE
C SUCC   =SUCCESSFUL MESSAGE NUMBER
C COL    =CURRENT COLLIDED PACKET NUMBER
C L      =INTERMEDIATE COLLISION RESOLUTION INTERVAL
C Q      =DUMMY VARIABLE EQUALS B
C NKK    =DUMMY VARIABLE SHOWS SUCCESS
C IR     =OUTPUT VECTOR OF LENGTH NR CONTAINING THE
C         UNIFORMLY DISTRIBUTED INTEGERS.
C S      =COLLISION RESOLUTION INTERVAL IN EACH STEP
C ED     =ENERGY DETECTOR
C ZK     =DUMMY VARIABLE
C T      =DUMMY VARIABLE
C LN     =COLLISION RESOLUTION INTERVAL
C
C *** VARIABLE DECLARATION
C
         INTEGER K,NR,B,SUM,SUCC,COLL,L,Q,XX,NKK
         INTEGER IR(1),S(20000),ED(30),LAST,ZK,T
         REAL*8 LN
         DOUBLE PRECISION DSEED
         DSEED=3.D0
         Z=15000
```

```
        DO 110 XX=1,1
        DO 111 Q=2,30
        LN=0
        L=0
        DO 234 A=1,Z
        B=XX
        SUCC=0
        K=Q
        COLL=K
        SUM=K
        ZK=1
 105    K=SUM
        IF (SUM.LE.XX) THEN
           B=SUM
        ENDIF
        NR=1
        DO 235 I=1,K
        CALL GGUD(DSEED,K,NR,IR)
        S(I)=IR(1)
  235   CONTINUE
        DO 301 AA=1,30
           ED(AA)=0
  301   CONTINUE
        DO 99 T=1,K
C ***  ENERGY DETECTOR
           GOTO(1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,
     *          18,19,20,21,22,23,24,25,26,27,28,29,
     *          30),S(T)
  1     ED(1)=ED(1)+1
        GOTO 99
  2     ED(2)=ED(2)+1
        GOTO 99
  3     ED(3)=ED(3)+1
        GOTO 99
  4     ED(4)=ED(4)+1
```

```
         GOTO 99
 5       ED(5)=ED(5)+1
         GOTO 99
 6       ED(6)=ED(6)+1
         GOTO 99
 7       ED(7)=ED(7)+1
         GOTO 99
 8       ED(8)=ED(8)+1
         GOTO 99
 9       ED(9)=ED(9)+1
         GOTO 99
 10      ED(10)=ED(10)+1
         GOTO 99
11       ED(11)=ED(11)+1
         GO TO 99
12       ED(12)=ED(12)+1
         GOTO 99
13       ED(13)=ED(13)+1
         GO TO 99
14       ED(14)=ED(14)+1
         GOTO 99
15       ED(15)=ED(15)+1
         GOTO 99
16       ED(16)=ED(16)+1
         GOTO 99
17       ED(17)=ED(17)+1
         GOTO 99
18       ED(18)=ED(18)+1
         GOTO 99
19       ED(19)=ED(19)+1
         GOTO 99
20       ED(20)=ED(20)+1
         GOTO 99
21       ED(21)=ED(21)+1
         GOTO 99
```

```
22      ED(22)=ED(22)+1
        GO TO 99
23      ED(23)=ED(23)+1
        GOTO 99
24      ED(24)=ED(24)+1
        GO TO 99
25      ED(25)=ED(25)+1
        GO TO 99
26      ED(26)=ED(26)+1
        GOTO 99
27      ED(27)=ED(27)+1
        GO TO 99
28      ED(28)=ED(28)+1
        GOTO 99
29      ED(29)=ED(29)+1
        GO TO 99
30      ED(30)=ED(30)+1
        GOTO 99
 99     CONTINUE
        DO 102 T=1,K
 102    CONTINUE
        SUM=0
        NKK=0
        DO 103 T=1,B
           IF ( ED(T).EQ.1) THEN
               SUCC=SUCC+1
               NKK=NKK+1
           END IF
           SUM=Q-SUCC
 103    CONTINUE
        L=L+B
        ZK=ZK+1
        IF (SUM.EQ.0) THEN
           IF (SUCC.EQ.COLL) GO TO 234
              SUM=COLL-SUCC
```

```
                    IF (SUM.LE.XX) GOTO 105
                       B=XX
                       GOTO 105
                     ENDIF
               GOTO 105
234        CONTINUE
           LN=L/Z
           WRITE (6,217) Q,XX,LN
           WRITE (8,217) Q,XX,LN
 111       CONTINUE
 110       CONTINUE
 217       FORMAT (' ', I3,5X,I3,5X,F12.4)
           STOP
           END
```

APPENDIX C

ANALYSIS PROGRAM FOR AMSCRA WITH SEPARATE RESOLUTION

```
$JOB
C *** THIS PROGRAM WAS WRITTEN TO CALCULATE THE L(N)
C     VALUES FOR THE AMSCRA WITH SEPARATE RESOLUTION.
C     STORAGE SHOULD BE AT LEAST 1M.
C
      INTEGER RR,B,I,J,BB,II,Z,G,FF,C,UL,Z1,K1
      INTEGER XX,SS,GG,QQ,KK,JJ
      REAL*8 KL(030,30),PP(030,030),LHAT(030),N
      REAL*8 PROB(31,31,31),SUM,SUB,MULT,LHATT2
      REAL*8 A(31,31,31),LHATT,L,M,P,VVV,ATZ,CR,ZS
      REAL*8 JJJ,PPP,LDENOM,LN,KKK,AAA,ZZ,TT,VV,L1L
      REAL*8 L2L,L12L,VBNO,VBOO
      PRINT 10
      PRINT 15,'N','B','L(N)'
      PRINT 16,'------------------------------'
   16 FORMAT(' ',02X,A30)
   15 FORMAT(' ',02X,A1,12X,A1,13X,A4)
   10 FORMAT ('1')
      KL(1,1)=0.0
      KL(2,1)=0.0
      PRINT 99,0,0,KL(1,1)
      PRINT 99,1,0,KL(2,1)
C
C ***** CALCULATION OF P(N,I)  ******************
C
C4          CONTINUE
C3    CONTINUE
      LHAT(1)=1.0
C
C*** A(B,J,I) COEFFICIENT *******
```

```
C
      DO 25 BB=1,31
         DO 26 JJ=1,31
            II=JJ
            WHILE (II.GE.1) DO
              IF (II.GT.JJ.OR.II.GT.BB) THEN
                 A(BB,JJ,II)=0.0
              ELSE IF (II.EQ.JJ.AND.II.EQ.BB) THEN
                 FACTI=FACTN(II-1)
                 A(BB,JJ,II)=FACTI
              ELSE IF (II.EQ.JJ) THEN
                 FACTB=FACTN(BB-1)
                 FACTBI=FACTN(BB-II)
                 A(BB,JJ,II)=FACTB/FACTBI
              ELSE
                 N=1
                 ZZ=BB
                 TT=JJ
                 VV=II
                 AAA=((ZZ-VV)**(TT-VV))
                 WHILE (N.LE.(JJ-II)) DO
                    AAA=AAA-A((BB-II+1),(JJ-II+1),N+1)
                    N=N+1
                 END WHILE
                 FACTB=FACTN(BB-1)
                 FACTJ=FACTN(JJ-1)
                 FACTI=FACTN(II-1)
                 FACTBI=FACTN(BB-II)
                 FACTJI=FACTN(JJ-II)
                 A(BB,JJ,II)=(1./FACTI)*FACTB*(1./FACTJI)
     *                            *(1./FACTBI)*FACTJ*AAA
               END IF
               IF (((JJ-1).NE.0).AND.(BB.GE.JJ)) THEN
                  CR=JJ
                  ATZ=(CR-1.)**(CR-1.)
                 PROB(BB,JJ,II)=(1./ATZ)*A(BB,JJ,II)
              END IF
                 II=II-1
           END WHILE
  26       CONTINUE
  25    CONTINUE
C
C ****CALCULATION OF L(N) ***
C
        DO 61 K=2,30
        DO 60 B=1,30
        IF (K.LE.B) THEN
C
C***** MESSAGE NUMBER LESS THAN OR EQUAL SLOT NUMBER
C***** USE  FIRST FORMULA
C
           SS=K
           SUM=0.0
           RR=SS-1
           DO 5  I=1,RR
              MULT=PROB(SS+1,SS+1,I+1)*LHAT(SS-I)
              SUM=SUM+MULT
 5         CONTINUE
           ZS=SS
           KL(SS,B)=(1./(1.-PROB(SS+1,SS+1,1)))*(ZS+SUM)
           LHAT(SS)=KL(SS,B)
           PRINT 99,SS,B,KL(SS,B)
        ELSE
C
C******* MESSAGE NUMBER GREATER THAN SLOT NUMBER ***
C******* USE SECOND FORMULA  ***********************
C
           M=K
           FACTK=FACTN(K)
           P=B
        L1L=0.0
        UL=MIN0(B,K)
        DO 31 I=1,UL
        J=K
          IF ((K-I).EQ.1) THEN
            LHATT=1.0
          ELSE
            LHATT=KL(K-I,B)
         ENDIF
         L1L=L1L+(A(B+1,J+1,I+1)/(P**K))*LHATT*((P/M)**K)
31     CONTINUE
       L2L=0.0
       K1=K-1
       DO 32 J=1,K1
         FACTJ=FACTN(J)
         FACTKJ=FACTN(K-J)
         VV=FACTK/(FACTJ*FACTKJ)
         Z=MIN0(B,J)
         Z1=Z+1
         DO 33 I=1,Z1
         IF ((J-I+1).EQ.0) THEN
            LHATT=0.0
         ELSE IF (J-I+1.EQ.1) THEN
            LHATT=1.0
         ELSE
            LHATT=KL(J-I+1,B)
         END IF
         IF ((K-J).EQ.1) THEN
            LHATT2=1.0
         ELSE
           LHATT2=KL(K-J,B)
         END IF
         L2L=L2L+(A(B+1,J+1,I)/(P**J))*VV*((P/M)**J)
    *        *((1.0-(P/M))**(K-J))*(LHATT+LHATT2)
33   CONTINUE
```

```
 32    CONTINUE
       L12L=B+L1L+L2L
       VBNO=(A(B+1,K+1,1)/(P**M))*((P/M)**K)
       VBOO=A(B+1,1,1)*((1.-(P/M))**K)
       LDENOM=1./(1.-VBNO-VBOO)
       LN=LDENOM*L12L
       KL(K,B)=LN
       PRINT 99,K,B,LN
       END IF
 60    CONTINUE
 61    CONTINUE
 99    FORMAT(' ',I3,10X,I3,10X,F8.4)
       STOP
       END
C      ****************************************************
C      *** FUNCTION SUBPROGRAM TO CALCULATE FACTORIAL N ***
C      ****************************************************
                    FUNCTION FACTN(N)
                    FACTN=1.
                    IF (N.GT.1) THEN
                        DO 2 I=2,N
                           FACTN=FACTN*I
 2                      CONTINUE
                    END IF
                    RETURN
                    END
$ENTRY
```

APPENDIX D

SIMULATION FOR AMSCRA WITH SEPARATE RESOLUTION

```
C ***THIS PROGRAM WAS WRITTEN TO MAKE THE SIMULATION OF THE
C    AMSCRA WITH SEPARATE RESOLUTION.
C ***VARIABLE DEFINITION ***
C K     = INPUT PAREMETER OF DISCRETE UNIFORM DISTRIBUTION
C          THE INTEGERS 1,2,...,K OCCUR WITH EQUAL
C          PROBABILITY. K MUST BE POSITIVE IN GGUD.
C B      =OBSERVATION INTERVAL.
C NR     =INPUT NUMBER OF RANDOM NUMBERS TO BEGENERATED.
C SUM    =DUMMY VARIABLE
C SUCC   =SUCCESSFUL MESSAGE NUMBER
C COL    =CURRENT COLLIDED PACKET NUMBER
C L      =INTERMEDIATE COLLISION RESOLUTION INTERVAL
C Q      =DUMMY VARIABLE EQUALS B
C NKK    =DUMMY VARIABLE SHOWS SUCCESS
C IR     =OUTPUT VECTOR OF LENGTH NR CONTAINING THE
C         UNIFORMLY DISTRIBUTED INTEGERS.
C S      =COLLISION RESOLUTION INTERVAL IN EACH STEP
C ED     =ENERGY DETECTOR
C ZK     =DUMMY VARIABLE
C T      =DUMMY VARIABLE
C LN     =COLLISION RESOLUTION INTERVAL
C ALAST  =SECOND PART OF THE COLLIDED PACKETS N-J
CSAKAT  = SAVED COLLIDED PACKETS GROUP
C
C *** VARIABLE DECLARATION
C
         INTEGER K,NR,B,SUM,SUCC,COLL,L,Q,XX,NKK,FL
         INTEGER IR(1),S(20000),ED(30),LAST,ZK,T
         INTEGER ALAST(100),SAKAT(100)
         REAL*8 LN
         DOUBLE PRECISION DSEED
         DSEED=3.D0
         Z=15000
         DO 110 XX=1,1
         DO 111 Q=2,30
C        XX=2
C        Q=4
         LN=0
         L=0
         DO 234 A=1,Z
         FL=1
         B=XX
         SUCC=0
         K=Q
         COLL=K
         SUM=K
         ZK=1
 105     K=SUM
         IF (SUM.LE.XX) THEN
            B=SUM
         ENDIF
         NR=1
         DO 235 I=1,K
         CALL GGUD(DSEED,K,NR,IR)
         S(I)=IR(1)
  235    CONTINUE
         DO 301 AA=1,30
            ED(AA)=0
  301    CONTINUE
         DO 99 T=1,K
            GOTO(1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,
     *            17,18,19,20,21,22,23,24,25,26,27,28,29,
     *            30),S(T)
  1      ED(1)=ED(1)+1
         GOTO 99
 2       ED(2)=ED(2)+1
         GOTO 99
 3       ED(3)=ED(3)+1
         GOTO 99
 4       ED(4)=ED(4)+1
         GOTO 99
 5       ED(5)=ED(5)+1
         GOTO 99
 6       ED(6)=ED(6)+1
         GOTO 99
 7       ED(7)=ED(7)+1
         GOTO 99
 8       ED(8)=ED(8)+1
         GOTO 99
 9       ED(9)=ED(9)+1
         GOTO 99
 10      ED(10)=ED(10)+1
         GOTO 99
11       ED(11)=ED(11)+1
         GO TO 99
12       ED(12)=ED(12)+1
         GOTO 99
13       ED(13)=ED(13)+1
         GO TO 99
14       ED(14)=ED(14)+1
         GOTO 99
15       ED(15)=ED(15)+1
         GOTO 99
16       ED(16)=ED(16)+1
         GOTO 99
17       ED(17)=ED(17)+1
         GOTO 99
18       ED(18)=ED(18)+1
         GOTO 99
19       ED(19)=ED(19)+1
```

```
        GOTO 99
20      ED(20)=ED(20)+1
        GOTO 99
21      ED(21)=ED(21)+1
        GOTO 99
22      ED(22)=ED(22)+1
        GO TO 99
23      ED(23)=ED(23)+1
        GOTO 99
24      ED(24)=ED(24)+1
        GO TO 99
25      ED(25)=ED(25)+1
        GO TO 99
26      ED(26)=ED(26)+1
        GOTO 99
27      ED(27)=ED(27)+1
        GO TO 99
28      ED(28)=ED(28)+1
        GOTO 99
29      ED(29)=ED(29)+1
        GO TO 99
30      ED(30)=ED(30)+1
        GOTO 99
 99     CONTINUE
        DO 102 T=1,K
 102    CONTINUE
        SUM=0
        NKK=0
        DO 103 T=1,B
           IF ( ED(T).EQ.1) THEN
               SUCC=SUCC+1
               NKK=NKK+1
           ELSE
              SUM=SUM + ED(T)
           END IF
 103    CONTINUE
        LAST=K-NKK-SUM
 106    CONTINUE
        SAKAT(FL)=K-LAST
        IF (LAST.NE.0) THEN
           ALAST(FL)=LAST
           FL=FL+1
        END IF
        L=L+B
        ZK=ZK+1
        IF (SUM.EQ.0) THEN
           IF (SUCC.EQ.COLL) GO TO 234
              SUM=ALAST(FL-1)
              FL=FL-1
              IF (SUM.LE.XX) GOTO 105
                 B=XX
                 GOTO 105
        ENDIF
        GOTO 105
234     CONTINUE
        LN=L/Z
        WRITE (6,217) Q,XX,LN
 111    CONTINUE
 110    CONTINUE
 217    FORMAT (' ', I3,5X,I3,5X,F12.4)
        STOP
        END
```

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